

COST ALLOCATION IN MANUFACTURING - REMANUFACTURING OPERATIONS

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Abstract

For firms remanufacturing their products, the total life-cycle costs and revenues from new and remanufactured products determine their profitability. In many firms, manufacturing/sales and remanufacturing/remarketing operations are carried out in different divisions. Each division is responsible for only part of the product's life cycle. Practices regarding transfer pricing across divisions vary significantly among companies, affecting the life-cycle profit performance of the product. In this research, we investigate the consequences of different types of transfer pricing approaches. To this end, we consider a manufacturer who also undertakes remanufacturing operations and we focus on price (quantity) decisions. In a two-period model, we analyze the profitability of different transfer pricing approaches. We determine that a cost allocation mechanism that allocates a portion of the initial production cost to each of the two stages of the product life-cycle is the most effective. We conclude with managerial recommendations concerning cost allocation.

Keywords: cost allocation, remanufacturing, closed-loop supply chain, transfer prices.

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1 Introduction

Remanufacturing is a process by which value is recovered from used products. During this process, used products are collected and disassembled; parts are tested and refurbished when feasible, and these parts are used in producing remanufactured products. Examples of remanufactured products include copiers, cell-phones, computers, automobile parts, printers, toner cartridges, etc. In this article, we focus on a manufacturer who sells a new product, and subsequently remanufactures it. This is a growing trend, accelerated by developments such as the WEEE Directive (Directive 2002/96/EC) that promotes producer responsibility.

Consumers typically value the remanufactured product less than the corresponding new product. In many countries, remanufactured products must be labeled as such, so the consumers are informed about their purchases. Therefore, a manufacturer/remanufacturer needs to make two distinct pricing decisions for the new and the remanufactured product. Since sales of new products determine the supply of remanufactured ones in the future, the two pricing decisions are linked. Frequently, the division responsible for the production/sales of the new products is different from the one in charge of remanufactured products. Consequently, the economic decisions regarding each are made by different managers, where the respective managers' performances are evaluated based on their division's profitability. Examples of such firms are easily found; they include Hewlett Packard (Guide and Van Wassenhove 2002), Bosch (Valenta 2004), DaimlerChrysler (Driesch and Flapper 2005) and Océ (Zuidwijk et al. 2005).

The natural question arising at this juncture is how the divisional profits should be measured so that each manager's incentives are aligned with those of the firm. Since the same inputs (materials, parts, etc.) may be used twice to generate revenues in different divisions, who should bear the cost of these inputs? Or, looking at it from the revenue perspective, should some of the revenue from remanufacturing be attributed to the manufacturing division since it contributed to generating that revenue? From the firm's perspective, it is overall profitability that matters, and different methods to distribute either cost or revenue will lead to different decisions by individual managers. This raises the following question: How do different methods compare in terms of total firm profit?

The mechanism that accounts for internal transfers of goods or services across different divisions within the firm, where the output from one division becomes an input into another, is transfer pricing. From both interviews and existing literature, we identified a wide variety of transfer pricing practices between the divisions responsible for manufacturing and remanufacturing. They range from essentially charging nothing to the remanufacturing division to charging the full standard cost of the good transferred. These approaches reflect a lack of consensus concerning the value of a used product. A non-exhaustive list of contrasting examples follow:

1. No manufacturing cost allocated to remanufacturing. In this approach, the manufacturing division bears the full cost of manufacturing the new product. Once the product is returned, the remanufacturing division is not charged any cost for using this input. This approach reflects the view that once the product is manufactured, the manufacturing cost becomes entirely sunk, therefore the firm should not take it into account in subsequent decisions.
2. Full manufacturing cost allocated to remanufacturing. In this approach, the remanufacturing division is charged the entire cost of the initial production, reflecting the view that the division is using a costly input. Interestingly, the manufacturing division has already been charged the same cost, so from the firm's perspective, this approach amounts to double counting.
3. Partial manufacturing cost allocated to remanufacturing. In this approach, an internal transfer price lower than the initial production cost is posted and the sales organization decides how many products to return for remanufacturing at the said price, and how many units to scrap (Zuidwijk et al. 2005).

Which of these approaches, if any, is appropriate? While there is an extensive economic and accounting literature on transfer pricing (Edlin and Reichelstein 1995, Vaysman 1998), this literature offers no guidance specific to closed-loop supply chains. In the operations literature, the only papers that raise a related issue - that of inventory valuation for remanufacturable products - are Teunter et al. (2000), Teunter (2001), and Teunter and van der

Laan (2002, 2004). These papers focus on setting the holding costs of returned products in decision support systems based on average cost models. They investigate whether the approximate average cost approach is the more suitable, compared to the discounted cash flow approach; and if so, what the “correct” holding costs for returned products should be. In other words, their focus is inventory valuation for centralized decision making. The operations literature on remanufacturing makes the implicit assumption that costs are known and optimal prices for the two products will be reached by the firm (Groenevelt and Majumder 2001a,b, Savaşkan et al. 2004, Debo et al. 2005, Ferrer and Swaminathan 2006). In practice, where different divisions and different decision makers are involved, the effectiveness of different transfer pricing methods becomes relevant.

Given the discord among existing transfer pricing approaches in practice, in conjunction with the lack of academic research, there is a clear need for specific transfer pricing guidelines in closed-loop supply chains. The goal of this paper is to develop such guidelines. To achieve this, we develop a model of a manufacturer who also undertakes remanufacturing operations; these operations are carried out in different divisions evaluated according to their respective divisional profits. We compare the performance of different transfer price mechanisms and identify one that aligns divisional incentives with those of the firm’s. This analysis yields guidelines specific to managing closed-loop supply chains.

We find that the optimal transfer price mechanism should allocate a portion of the initial production cost to each of the two stages in the product life-cycle. Initial production can be seen as an investment in both the new product and the remanufactured product. While its cost is traditionally borne by the manufacturing division, its benefits are reaped by the remanufacturing division as well. Compensating the manufacturing division for accessing used products ensures that divisional incentives are aligned with those of the firm’s.

One important feature of our transfer pricing mechanism is that the charges made to the remanufacturing division are “fixed.” In other words, the remanufacturing division needs to pay a “fixed” fee in order to benefit from the initial production of the new product. By “fixed,” we mean that charges are determined before the quantity of units transferred is set, therefore it seems fixed to the division managers, who cannot influence it through their own decisions. By paying this charge, the downstream division obtains the right for

future transfers. This way, we avoid double marginalization, which is a common pitfall of transfer prices. The transfer pricing literature would predict that a fixed charge is sufficient for the downstream division to reach the optimal decisions for the firm. Our results provide a specific answer to what this fixed sum should be to align the upstream division's initial production decision. This has not been documented previously, but was brought out nicely in the remanufacturing framework.

The remainder of the paper is organized as follows: In Section 2, we describe the model and establish the firm-wide optimal benchmark. Section 3 investigates the effectiveness of different transfer pricing mechanisms and establishes a specific cost allocation mechanism as achieving the benchmark profit level. The robustness of our results is discussed in Section 4. We conclude with the managerial implications of our findings in Section 5.

2 The Model

In this section, we present the model used to answer the research questions posed above.

Firm Structure. Modern firms of a certain size tend to be decentralized, i.e., decision rights are delegated from the headquarters to divisional managers, when the economic environment is too complex and making decisions centrally may no longer be efficient. As discussed in the introduction, a decentralized structure is prevalent in firms that undertake both manufacturing and remanufacturing operations. In our model, one division, D1, incurs the manufacturing cost and determines the new product's sales price. Revenues from the new product sales accrue to this division, whose operations encompass both manufacturing and sales activities for the new product. Alternatively, one could view this division as the new-product sales organization, which is charged the unit standard manufacturing cost for each new unit produced by manufacturing. The downstream division, D2, remanufactures some used products and salvages the rest. Remanufactured product sales and used product salvage revenues accrue to this division. The respective managers of these divisions will be referred to as M1 and M2.

Performance Evaluation. The literature on executive compensation documents executive

salaries as having four main components: base salaries, annual bonus plans, stock options and others (Murphy 1998). Among these components, annual bonus plans are put in place primarily for incentive purposes. The precise structure of these bonus plans vary from firm to firm, but they all invariably use some accounting measures of profits to measure performance. For incentive purposes, the divisional profit is the most important in evaluating the divisional manager's performance and compensating him/her accordingly. In the accounting literature, this is called responsibility accounting, where the appropriate performance measure should evaluate the results within the domains of the manager's responsibility. For these reasons, we focus on the divisional profit as the performance measure for divisional managers. The method used by the firm to compute the divisional profits, a part of the internal accounting system, then becomes crucial.

Market Characteristics. We assume that the new and remanufactured products are sold to different market segments that do not overlap. This is appropriate when the remanufactured product is sold in a different geographic region, or when the customers in these segments have distinct preferences. Specifically, firms can limit cannibalization by selling through different channels (Guide and Van Wassenhove 2002), or by remanufacturing only a subset of their product line where they believe cannibalization effects are limited (Valenta 2004).

Product life-cycle. We assume that the new product can be used for only one period. Subsequently, products are returned by customers. A fraction $q \in [0, 1]$ of returned products can be remanufactured; the rest cannot be used and will be scrapped. Remanufactured products also have a one-period lifetime, after which they are disposed of. Similar assumptions are adopted in other papers, e.g., Groenevelt and Majumder (2001a,b), Debo et al. (2005) and Ferguson and Toktay (2006). With the above assumptions, it is sufficient to analyze a two-period model. Only new products are sold in the first period by D1, and only remanufactured products are sold in the second period by D2.

Consumer Characteristics. We assume that consumer willingness-to-pay is heterogeneous and uniformly distributed in the interval $[0, 1]$. The market size is normalized to 1. Consumers typically value the remanufactured product less than the new product. We model this by letting a consumer of type $\phi \in [0, 1]$ have a willingness-to-pay of ϕ for a new product

and $(1 - \delta)\phi$ for a remanufactured product. The parameter δ is “perceived depreciation” (Debo et al. 2005). The utility each consumer derives from purchasing a product is equal to the difference between his willingness-to-pay and the product price.

We let p_n and d_n denote the (first-period) new product price and demand, and p_r and d_r the (second-period) remanufactured product price and demand. With the consumer utility function defined as above, the following linear inverse demand functions are obtained:

$$p_n = 1 - d_n \quad (1)$$

$$p_r = (1 - \delta)(1 - d_r). \quad (2)$$

The Cost Structure. We denote the unit manufacturing cost by c ; and the unit remanufacturing cost by c_r . Second-period profits are discounted by $\beta < 1$. Returned remanufacturable products have salvage value s per unit. The disposal cost for nonremanufacturable units is assumed to be zero.

We make the following assumptions: (i) $0 \leq c - \beta sq \leq 1$, (ii) $0 \leq c_r \leq (1 - \delta) - s$, and (iii) $s < 1 - \delta$. These assumptions ensure that the optimal firm-wide solution satisfies the following properties: $0 < d_n^* < \infty$ and $d_r^* > 0$.

We now establish firm-wide profit maximizing decisions as benchmarks for our later results. Because of the one-to-one correspondence between price and demand, the optimization problem can be formulated either in terms of prices or demands. We proceed with the latter for ease of interpretation.

The optimal demand levels are those that maximize the present value of the profits from both periods subject to an availability constraint:

$$\max_{d_n, d_r} d_n p_n(d_n) - c d_n + \beta (d_r p_r(d_r) - c_r d_r + s(q d_n - d_r)) \quad \text{s.t. } q d_n - d_r \geq 0. \quad (3)$$

Lemma 1 *The optimal demand levels for new and remanufactured products are*

$$(d_n^*, d_r^*) = \begin{cases} \left(\frac{1-c+\beta sq}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)} \right) & \text{if } c_r + s \geq (1 - q(1 - c + \beta sq))(1 - \delta) \\ \left(\frac{1-c+\beta q(1-\delta-c_r)}{2+2\beta q^2(1-\delta)}, q d_n^* \right) & \text{otherwise.} \end{cases} \quad (4)$$

All proofs can be found in the Appendix. In the first case, the quantity of products remanufactured is unconstrained by the availability of remanufacturable products from past

sales; some remanufacturables are salvaged. In the second case, the firm prefers to remanufacture more units were there any, but is constrained by the amount available. By examining the inequality in (4), we identify conditions driving each case. In particular, remanufacturing may become constrained by availability when the remanufacturing cost, the salvage value or the perceived depreciation is low. All these conditions increase the appeal of remanufactured products, making the constraint binding. Similarly, when manufacturing cost is high, obtaining revenues through remanufacturing becomes more attractive.

3 Cost Allocation Methods

In practice, a variety of cost allocation methods are used in firms, reflecting different attitudes concerning the value of a used product. We now establish firm profits with the various cost allocation methods mentioned in the introduction and discuss their effectiveness. Recall that decisions about the new and remanufactured products are made by different managers M1 and M2. Their respective compensation is strictly increasing in their divisional profits, which they strive to maximize.

3.1 Full manufacturing cost allocated to D1.

In this case, M1 maximizes $(p_n - c)d_n$ and M2 maximizes $(p_r - c_r)d_r + s(qd_n - d_r)$, s.t. $d_r \leq qd_n$, where d_n has been chosen by M1.

Lemma 2 *With manufacturing cost allocated to only to D1, the demand levels are*

$$(\hat{d}_n, \hat{d}_r) = \begin{cases} (\frac{1-c}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)}) & c_r + s \geq (1 - q(1 - c))(1 - \delta) \\ (\frac{1-c}{2}, q\frac{1-c}{2}) & \textit{otherwise.} \end{cases} \quad (5)$$

There are several observations from comparing the decisions reached in (4) and (5). First, consider the case where the second period's production is *unconstrained* in both scenarios. It is easy to see that \hat{d}_n is less than d_n^* ; the difference is $\frac{\beta sq}{2}$. Division 1's profit only improves when a new product is made and sold; M1 does not benefit from the salvage value that can be obtained at the end of life. In contrast, both revenues are valuable to the firm. M1, therefore, under-produces. One can see that in the second period, the costs and revenues

accruing to the firm are the same as those accruing to M2. Consequently, M2 applies the same decision rule as the firm, so that $\hat{d}_r = d_r^*$. Note this equality only holds if $\hat{d}_r \leq q\hat{d}_n$. The threshold for this constraint to be binding is higher under the decentralized setting: $(1 - q(1 - c))(1 - \delta) \geq (1 - q(1 - c + \beta sq))(1 - \delta)$. In other words, the remanufactured product quantity is constrained for a larger set of parameters. This is a second-order effect of the under-production of the new product.

When $c_r + s < (1 - q(1 - c))(1 - \delta)$, the demand levels in both periods are suboptimal. In this case, any additional new product produced will be remanufactured by M2. Similarly as in the unconstrained case, M1 ignores the effect of his decision on future operations. In particular, he ignores the benefit of future revenues from remanufacturing, since he does not reap any benefit from this activity. Consequently, he under-produces, leading M2 to under-produce as well. These observations are formalized in the following proposition.

Proposition 1 *Assume $s > 0$. With manufacturing cost allocated only to D1, M1 always under-produces and M2 under-produces when $c_r + s < (1 - q(1 - c))(1 - \delta)$.*

$$\begin{aligned} \hat{d}_n < d_n^*, \quad \text{and} \quad \hat{d}_r = d_r^* & \quad \text{if} \quad c_r + s \geq (1 - q(1 - c))(1 - \delta) \\ \hat{d}_n < d_n^*, \quad \text{and} \quad \hat{d}_r < d_r^* & \quad \text{if} \quad c_r + s < (1 - q(1 - c))(1 - \delta). \end{aligned}$$

The only situation when there is no under-production of new products is for those scenarios with negligible salvage value ($s = 0$), and d_r unconstrained by d_n . This captures the fact that subsequent to incurring the initial cost c during production, this cost becomes entirely sunk for the remanufacturing operation. In other words, given zero salvage value, one cannot recoup any economic value from the returns. In conjunction with the fact that M2 reaches the optimal decision in this case, we conclude that this cost allocation method is appropriate in this special case. Consequently, for processes where products have negligible salvage value, and remanufacturing is not constrained by availability, the firm does not have to coordinate incentives between divisional managers.

In a case we observed in practice, the full manufacturing cost was also charged to D2. In this case, M1 maximizes $(p_n - c)d_n$ and M2 maximizes $(p_r - c_r)d_r + s(qd_n - d_r) - cd_n$, s.t. $d_r \leq qd_n$, where d_n has been chosen by M1. Because cd_n is not affected by M2's actions, the solution is the same as in (5). However, the profits of D2 are reduced by cd_n , possibly

putting this division in the red, and putting in question its economic viability. While it is easy to observe that double counting is not a productive approach, we mention it here explicitly to point out the potential detrimental effect of fully allocating the manufacturing cost to the remanufacturing division.

3.2 Manufacturing cost shared between manufacturing and re-manufacturing.

Motivated by the partial cost allocation case we observed in practice, we study a cost allocation scheme where a fraction α of the unit manufacturing cost is charged to D1, and the rest, $1 - \alpha$, to D2, for each unit manufactured by D1. Thus, M1 maximizes $(p_n - \alpha c)d_n$ and M2 maximizes $-(1 - \alpha)cd_n + d_r p_r(d_r) - c_r d_r + s(qd_n - d_r)$. s.t. $d_r \leq qd_n$, where d_n has already been chosen by M1. Note that differing from the case we reported (Zuidwijk et al. 2005), the amount $(1 - \alpha)cd_n$ charged to D2 is fixed; it does not depend on a quantity decision by either division about how many used products to sell to or buy from each other.

Proposition 2 *In the presence of cost allocation scheme $\{\alpha, 1 - \alpha\}$, the demand levels are*

$$(\hat{d}_n, \hat{d}_r) = \begin{cases} \left(\frac{1-\alpha c}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)} \right) & \text{if } c_r + s \geq (1 - \delta)(1 - q(1 - \alpha c)) \\ \left(\frac{1-\alpha c}{2}, \frac{1-\alpha c}{2}q \right) & \text{otherwise.} \end{cases} \quad (6)$$

Note that setting $\alpha = 1$ recovers the analysis in §3.1. We now investigate whether there is a specific value of α that yields the firm-wide benchmark profit.

Proposition 3 *In the presence of cost allocation scheme $\{\alpha, 1 - \alpha\}$, optimal decisions can be reached when*

$$\alpha^* = \begin{cases} 1 - \frac{\beta s q}{c} & \text{if } c_r + s \geq (1 - q(1 - c + \beta s q))(1 - \delta) \\ \frac{c + \beta q(c_r + (1 - \delta)(q - 1))}{c(1 + \beta q^2(1 - \delta))} & \text{otherwise.} \end{cases}$$

With this allocation scheme,

$$(\hat{d}_n, \hat{d}_r) = \begin{cases} \left(\frac{1-c+\beta s q}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)} \right) & \text{if } c_r + s \geq (1 - q(1 - c + \beta s q))(1 - \delta) \\ \left(\frac{1-c+\beta q(1-\delta-c_r)}{2+2\beta q^2(1-\delta)}, q\hat{d}_n \right) & \text{otherwise.} \end{cases}$$

Note that under the cost allocation scheme α^* , we obtain demand levels that are exactly the same as the benchmark demand levels (d_n^*, d_r^*) . In other words, we have shown that a proportional cost allocation scheme with α not necessarily equal to 1 is optimal.

The logic behind the expression for α^* is not immediately apparent. We interpret this result next. Let $C(d_n)$ denote the effective cost to the firm of producing d_n units.

Proposition 4 $\alpha^*c = \frac{\partial C}{\partial d_n}|_{d_n=d_n^*}$.

This proposition states that in both cases of Proposition 3, the expression obtained for α^* is such that the cost allocated to D1 equals the marginal cost to the firm evaluated at $d_n = d_n^*$.

Let us discuss $\alpha^* = 1 - \frac{\beta sq}{c}$, the case with ample supply of used remanufacturable products; the other case follows the same logic. Rewriting α^*c as $c - \beta sq$, we see that this quantity is the marginal cost of producing a new product to the entire firm when the next unit will be salvaged (since there is already sufficient supply).

When we allocate only α^* portion of the production cost to M1 in the first period, he makes the following calculation:

$$\max_{d_n} d_n(1 - d_n) - \alpha^*cd_n.$$

Differentiating with respect to d_n , we obtain

$$\underbrace{1 - 2d_n}_{\text{marginal revenue}} - \underbrace{\frac{c - \beta sq}{c}c}_{\text{firm's marginal cost}} = 0,$$

which yields $d_n^* = \frac{1 - c + \beta sq}{2}$, the optimal production quantity of new products. M1 always chooses the quantity that makes his marginal revenue equal to his marginal cost. His marginal revenue is already equal to that of the firm's. By changing his marginal cost from c to α^*c , we are able to incorporate the firm-wide incentives into his performance measure.

For the other case, α^*c can again be interpreted as the marginal cost of producing one more new product at d_n^* , but this time when the next unit would be remanufactured. Again, by charging α^*cd_n , we are substituting the firm's incentives into M1's performance measure, and thereby incorporating the firm's tradeoffs into M1's decision making.

We make three other observations about the structure of the cost allocation scheme:

1. *The production cost should be shared between manufacturing and remanufacturing operations.* In other words, in a closed-loop supply chain, the production cost should not be borne only by the manufacturing division; a portion of the unit production cost should be allocated to each division that benefits from it. It is not currently the standard practice to allocate the production cost to any other division than manufacturing. The status quo is driven by characteristics of forward supply chains, but is not appropriate for closed-loop supply chains. This is because in closed-loop supply chains, value accrues to the firm at different junctions of the life-cycle of the product, and via operations carried out in different divisions. To reflect this, the production cost needs to be shared by the participating divisions if firm-wide optimality is to be reached.

2. *Cost allocation should be implemented as a fixed cost allocation.* In other words, charges should be determined independently of the *actual* quantity of units remanufactured, and be based on a predetermined level. By paying this charge, the downstream division obtains the right for future transfers. This way, double marginalization is avoided, which is the usual pitfall of transfer prices. In the above formulation, this mechanism is captured by letting the first term in M2's objective function be $(1 - \alpha)cd_n$, and not $(1 - \alpha)cd_r$. Zuidwijk et al. (2005) describe a case where the allocated cost is unitized instead. Our result says that this practice will lead to sub-optimal decisions and should be avoided.

3. *Decentralization maintains optimality in some cases.* With no salvage value and sufficient supply, the decentralized solution is optimal without cost allocation (If $s = 0$, then $\alpha^* = 1$ in the unconstrained case). In this case, there is no future value to the firm from manufacturing one additional unit. Since all the benefit accrues to the firm from the manufactured product, all the cost must be allocated to this stage.

4 Discussion

The goal of this paper is to provide some guidelines concerning cost allocation methods in closed-loop supply chains. To this end, we developed and analyzed a parsimonious model,

focusing on characteristics of the remanufacturing setting. In this section, we comment on the validity of revenue-based approaches and the robustness of our results to some of our assumptions.

Revenue allocation. Since the inputs used by D1 yield value a second time in remanufactured products, another approach that may appear promising is to allocate part of the revenue from remanufacturing to D1, but this does not work for the following two reasons. Such a method induces underproduction by the remanufacturing division since this division does not obtain all the benefits from remanufacturing. Its incentives are therefore mis-aligned with those of the firm on the margin. It is also ineffective in inducing D1 to manufacture the optimal level of new products. Consider the case where the marginal unit produced would be salvaged (case i). Cost allocation works by making the marginal profit calculation of D1 the same as the firm's at the optimal production quantity:

$$\underbrace{1 - 2d_n}_{\text{marginal revenue}} - \underbrace{c - \beta sq}_{\text{firm's marginal cost}} = 0,$$

We can alternatively view βsq not as a term that reduces cost, but increases revenue (for the firm and for D1):

$$\underbrace{1 - 2d_n + \beta sq}_{\text{marginal revenue}} - \underbrace{c}_{\text{firm's marginal cost}} = 0,$$

Therefore, the cost allocation mechanism can be viewed as one that increases the revenue of D1 by βsq for each unit manufactured. No revenue allocation scheme that is proportional to D2's decision d_r can achieve this since D1 needs to obtain this revenue for each unit it manufactures.

Demand structure. We considered a simple demand function of the form $p_n = 1 - d_n$. This function can be generalized to $p_n = a - bd_n$. In this case, the same cost allocation structure $\{\alpha, 1 - \alpha\}$ is optimal with

$$\alpha^* = \begin{cases} 1 - \frac{\beta sq}{c} & \text{if } c_r + s \geq (a - q(a - c + \beta sq))(1 - \delta) \\ \frac{c + \beta q(c_r + a(1 - \delta)(q - 1))}{c(1 + \beta q^2(1 - \delta))} & \text{otherwise.} \end{cases}$$

We observe an interesting robustness result. When the supply of used products is sufficient, the proportion of production cost allocated to the two divisions is independent of the demand parameters. Recall that the cost allocation mechanism works by making D1's marginal cost

the same as the firm's. When supply is adequate, the effective marginal cost of producing one more new product depends only on its salvage value at end of use, and not on the demand for remanufactured products. When supply is constrained, however, the effective marginal cost of producing one more unit depends on the remanufactured product demand/price. In that case, the cost allocation mechanism does depend on the specific demand function. However, the dependence is through only one parameter, a , so determining only this demand parameter is sufficient for cost allocation purposes.

Disposal cost. We assumed the disposal cost to be 0. Suppose that the unit disposal cost of unusable products is $v > 0$. In the firm-wide optimal benchmark, fewer units are manufactured because the total margin from new production is reduced. In the decentralized case, the new product volume depends on which party is charged the cost of disposal.

If D1 is charged this cost, the underproduction relative to the optimal solution remains unchanged because underproduction occurs only due to the omission of the future income from salvaging unused remanufacturable products. In this case,

$$\alpha^* = \begin{cases} 1 - \frac{\beta sq}{c} & \text{if } c_r + s \geq (1 - q(1 - c + \beta(sq - v(1 - q)))(1 - \delta)) \\ \frac{c + \beta q c_r + \beta q(1 - \delta)(q - 1)(1 + v\beta q)}{c(1 + \beta q^2(1 - \delta))} & \text{otherwise.} \end{cases}$$

Similar to the discussion above, when the supply of used products is sufficient, the proportion of production cost allocated to the two divisions is independent of the disposal cost. This is again because in the case of ample supply, the actions of the remanufacturing division are not impacted by those of the manufacturing division.

If D2 is charged the disposal cost, the new product volume is independent of this cost. And the remanufacturing division's choice of remanufacturing quantity is aligned with those of the firm as before. Therefore, the structure and value of α^* remains unchanged from the case with no disposal cost.

Salvage value. We assume the salvage value of usable products to be positive, since there may be a market for these products even without remanufacturing. However, this quantity could be negative if there is a cost to dispose of these products. In this case, the underproduction results relating to new products will change to over-production, but the structure of the results and the insights pertaining to the optimal cost allocation mechanism will remain unchanged.

In §3.1, we had noted that if $s = 0$ and supply is not constrained, decentralization without cost allocation is optimal. In other words, transfer pricing is not of particular concern when the salvage value of the new product is low or when there is a plentiful supply of used products. Based on the discussion in the previous two items, we conclude that this observation is robust to the demand structure and to the existence of disposal cost.

Information structure. Our paper does not explicitly address the question of asymmetric information in a delegation setting. A related study by Dutta and Reichelstein (2002) examines a model where the managers have private information regarding the production function. They demonstrate that it is optimal for the principal to delegate the investment decision and to base the manager’s compensation on the residual income performance measure. In our model, the divisional profit function is precisely a form of the “residual income;” like in their paper, the performance measure for managers in our model is expressed as divisional profit minus any cost charges assigned to the division by the headquarters. Consistent with Dutta and Reichelstein’s findings, should there be asymmetric information between the divisional managers and headquarters, the performance measure we use, and in turn our proposed cost allocation method, would still be the one achieving the highest possible profit attainable given the agency cost of information asymmetry, leading to second-best outcomes for the firm (by definition).

Cost accuracy. Our analysis assumes that costs are accurate and certain. In practice, cost figures often contain estimation errors and are subject to much debate. Estimation errors can be minimized through an appropriate application of activity-based costing (commonly known as ABC), which is a process by which firms arrive at their cost parameters. Since we take cost parameters as given, our results are robust to the firm’s choice of a costing system.

5 Conclusions

In firms where manufacturing and remanufacturing operations are performed in different divisions, their managers’ performances are often evaluated separately. Yet the economics of new and remanufactured products are linked; the output of manufacturing eventually becomes the input to remanufacturing. In decentralized firms, the system that accounts for

internal transfers of goods across different divisions is transfer pricing. We were intrigued by observations of widely different transfer prices between the manufacturing and remanufacturing divisions in practice. This paper examines the effectiveness of different transfer pricing methods and make recommendations concerning their use in closed-loop supply chains.

For our analysis, we develop a basic model that captures some important characteristics of manufacturing/remanufacturing operations. We develop a cost allocation scheme that achieves the optimal firm-wide solution. Our results provide several recommendations that can assist in coordinating the manufacturing and remanufacturing operations in practice, by aligning the incentives of all the parties concerned. We also identify features of closed-loop supply chains that render direct application of results from forward supply chains unsuitable. These findings are robust to assumptions on demand structure, disposal cost, salvage value and cost accuracy. They are discussed below.

First, a portion of the initial production cost should be allocated to the remanufacturing division, and not be borne solely by the manufacturing division. This may sound counterintuitive at first, given that once the production costs are incurred, they become sunk. This raises the question of why the manufacturing cost should be allocated, as our results suggest. The reason is that in a closed-loop supply chain, value accrues to the firm at different points in the life-cycle of the product and via operations of different divisions. Allocating a portion of the unit production cost to each division that benefits from it ensures each division's incentives are aligned with those of the firm. It is not currently standard practice to allocate the production cost to any other division than manufacturing. However, existing practice is driven by characteristics of forward supply chains, and is not appropriate for closed-loop supply chains.

While novel in the literature, this result does not run counter to findings from existing literature. In particular, it is consistent with the intuition from the literature on sharing the cost of capital investments such as new facilities, machines or technologies. The entire firm may benefit from such an investment, while one division incurs the investment cost. Or, while the investment is incurred in the current period, its benefits accrue over several years. The problem in these cases is that the division making the investment tends to underinvest, either because it does not reap the entire benefit of the investment, or because the gains are

accrued over time and the current period profits suffer. In the literature, the issue is how to align incentives for the division making the investment in a way that is optimal for the entire firm. This is done by allocating part of the cost of the investment to other divisions (Wei 2004) or spreading it over time to reflect its pattern of future potential benefits. These allocation schemes are developed for capital investments; this concept has not been previously proposed on a production cost basis. In a closed-loop supply chain, the initial cost of the new product can be viewed as an investment whose benefit will be reaped later and/or by another party, so the same principle applies.

Second, we identify cases where the logic derived from forward supply chains is sufficient: If the salvage value of a product is negligible or there are enough used products that some are salvaged, decentralization is not an issue. This is because an additional unit of production has no future revenue impact. Therefore, it is appropriate to charge the entire production cost to the manufacturing division.

Third, if part of the initial production cost is allocated to other divisions, it must be allocated so it is a fixed cost; the allocated cost must not be unitized. In other words, the charge should be independent of the quantity remanufactured later and apply to all items produced by the manufacturing division. Doing otherwise would introduce double marginalization in the remanufacturing division, leading to a lower level of remanufacturing than is optimal for the firm. This is in line with the transfer pricing literature, according to which charging the downstream division a fixed cost should be sufficient to attain the firm-wide optimal solution. Our results provide a specific answer as to how this fixed sum should be determined in order to provide proper incentives to the division managers to maximize firm profits.

Finally, revenue sharing - allocating part of the revenue from remanufacturing back to manufacturing - is undesirable as it induces underproduction at the remanufacturing stage, and is ineffective at providing proper incentives to the manufacturing division to produce the optimal number of new products.

The salvage value of a remanufacturable product is its value on the secondary market if sold as is after use. Some products (e.g. servers, automotive parts) have a high salvage value because there are well-developed secondary markets for them. Other products have

a relatively low value on the secondary market or are typically scrapped, but can have a relatively high value if remanufactured, such as truck tires (Debo et al. 2005), or remanufactured in a timely fashion, such as PCs, printers, cell phones (Blackburn et al. 2004, Souza et al. 2005). Our results say that proper cost allocation is especially important for remanufacturable products with a high salvage value. It can also be important for remanufacturable products where the salvage value is low, if the supply of used remanufacturable products is limited.

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Appendix

Proof of Lemma 1. The Hessian of the objective function in (3) is $\begin{pmatrix} -2 & 0 \\ 0 & -2(1-\delta) \end{pmatrix}$, whose leading coefficient is negative and whose determinant $4(1-\delta)$ is positive since we assumed $\delta < 1$. Thus, the Hessian is negative definite and the profit function is strictly concave in (d_n, d_r) . The associated Lagrangean is

$$L(d_n, d_r, \lambda) = d_n p_n(d_n) - c d_n + \beta(d_r p_r(d_r) - c_r d_r + s(q d_n - d_r)) + \lambda(q d_n - d_r).$$

The triplet $(d_n^*, d_r^*, \lambda^*)$ satisfy the Kuhn-Tucker conditions

$$\frac{\partial L}{\partial d_n} = p_n + d_n p_n' - c + \beta s q + q \lambda = 1 - 2d_n - c + \beta s q + q \lambda = 0 \quad (7)$$

$$\frac{\partial L}{\partial d_r} = \beta(p_r + d_r p_r' - c_r - s) - \lambda = \beta((1-\delta)(1-2d_r) - c_r - s) - \lambda = 0 \quad (8)$$

$$\lambda(q d_n - d_r) \geq 0, \lambda \geq 0.$$

Here, we used $p_n = 1 - d_n$, $p_r = (1 - \delta)(1 - d_r)$, $p_n' = -1$ and $p_r' = -(1 - \delta)$.

Case *i*. Assume that $q d_n^* - d_r^* > 0$. Then $\lambda^* = 0$. From (7), we obtain $d_n^* = \frac{1-c+\beta s q}{2}$; and from (8), we obtain $d_r^* = \frac{1-\delta-c_r-s}{2(1-\delta)}$. We now need to check whether $q d_n^* - d_r^* > 0$ holds. Substituting and simplifying, we find that this condition holds if $c_r + s > (1 - \delta)(1 - q(1 - c + \beta s q))$. If $c_r + s = (1 - \delta)(1 - q(1 - c + \beta s q))$, then $q d_n^* - d_r^* = 0$ and $\lambda^* = 0$.

Case *ii*. If $c_r + s < (1 - \delta)(1 - q(1 - c + \beta sq))$, then λ^* cannot be 0; $\lambda^* > 0$ must hold. In this case, $qd_n^* = d_r^*$. To solve for d_n^* , solve for λ in (8) and plug it into (7): $1 - 2d_n^* - c + q\beta((1 - \delta)(1 - 2qd_n^*) - c_r) = 0$. Then $d_n^* = \frac{1+q\beta(1-\delta-c_r)-c}{2(1+q^2\beta(1-\delta))}$. To check that $\lambda^* > 0$, we substitute d_n^* into (7), which yields the condition $c_r + s < (1 - \delta)(1 - q(1 - c + \beta sq))$. This is what we had assumed, so we are done. ■

Proof of Lemma 2. In period one, the profits are

$$\Pi_1 = d_n p_n - c d_n = d_n(1 - d_n) - c d_n.$$

The demand level that maximizes period 1's profit, denoted by \hat{d}_n , fulfills the following condition:

$$\begin{aligned}\Pi_1'(\hat{d}_n) &= 1 - 2\hat{d}_n - c = 0 \\ \hat{d}_n &= \frac{1 - c}{2}.\end{aligned}$$

In period two, the profit has the following profit:

$$\begin{aligned}\Pi_2 &= d_r p_r - c_r d_r + s(q\hat{d}_n - d_r) \\ &= d_r(1 - \delta)(1 - d_r) - c_r d_r + s(q\hat{d}_n - d_r) \quad \text{s.t. } d_r \leq q\hat{d}_n.\end{aligned}$$

$\Pi_2' = (1 - \delta)(1 - 2d_r) - (c_r + s)$ and $\Pi_2'' = -2(1 - \delta)$. Therefore, Π_2 is strictly concave in d_r , and is maximized at $d_r = \frac{1-\delta-c_r-s}{2(1-\delta)}$. Taking the constraint into account, we conclude that $\hat{d}_r = \min(q\hat{d}_n, \frac{1-\delta-c_r-s}{2(1-\delta)})$. Using $\hat{d}_n = \frac{1-c}{2}$, the two cases can be rewritten as follows:

Case (i): If $c_r + s \geq (1 - q(1 - c))(1 - \delta)$, the second-period solution is unconstrained and $\hat{d}_r = \frac{1-\delta-c_r-s}{2(1-\delta)}$.

Case (ii): If $c_r + s < (1 - q(1 - c))(1 - \delta)$, then the supply of recovered goods becomes a constraint and $\hat{d}_r = q\hat{d}_n = q\frac{1-c}{2}$. ■

Proof of Proposition 1.

Case (i): Let $c_r + s \geq (1 - q(1 - c))(1 - \delta)$. Comparing Lemmas 1 and 2, it is easy to see that $d_n^* = \frac{1-c+\beta sq}{2} > \frac{1-c}{2} = \hat{d}_n$ for $s > 0$, and that $d_r^* = \hat{d}_r$.

Case (ii): Let $(1 - q(1 - c + \beta sq))(1 - \delta) \leq c_r + s < (1 - q(1 - c))(1 - \delta)$. d_n^* and \hat{d}_n remain the same. We need to check whether $d_r^* > \hat{d}_r$, i.e., whether $\frac{1-\delta-c_r-s}{2(1-\delta)} > q\frac{1-c}{2}$. This condition is equivalent to $c_r + s < (1 - q(1 - c))(1 - \delta)$, so we are done.

Case (iii): Let $c_r + s < (1 - q(1 - c + \beta sq))(1 - \delta)$. For $s > 0$, this inequality implies $c_r < (1 - q(1 - c))(1 - \delta)$, which can be rewritten as $(1 - \delta)(q - cq) < (1 - \delta - c_r)$. Also note that

$$\begin{aligned}\hat{d}_n &= \frac{1 - c}{2} \\ &= \frac{(1 - c)(1 + \beta q^2(1 - \delta))}{2(1 + \beta q^2(1 - \delta))} \\ &= \frac{1 - c + \beta q(1 - \delta)(q - cq)}{2 + 2\beta q^2(1 - \delta)} \\ &< \frac{1 - c + \beta q(1 - \delta - c_r)}{2 + 2\beta q^2(1 - \delta)} = d_n^*,\end{aligned}$$

where we use this inequality in the last step. We conclude that $qd_n^* > q\hat{d}_n$, which implies $d_r^* > \hat{d}_r$ for this case. ■

Proof of Proposition 2. The maximum profit in period 1 is

$$\Pi_1(\alpha) = \max d_n p_n(d_n) - \alpha c d_n = \max d_n(1 - d_n) - \alpha c d_n. \quad (9)$$

The optimal solution is $\hat{d}_n = \frac{1 - \alpha c}{2}$. The corresponding price is $\hat{p}_n = 1 - \hat{d}_n = \frac{1 + \alpha c}{2}$ and the corresponding profit is $\Pi_1(\alpha) = \frac{(1 - \alpha c)^2}{4}$.

The maximum profit in period 2 is

$$\begin{aligned}\Pi_2(\alpha) &= -(1 - \alpha)c\hat{d}_n + \max_{d_r \leq q\hat{d}_n} d_r p_r(d_r) - c_r d_r + s(q\hat{d}_n - d_r) \\ &= -(1 - \alpha)c\hat{d}_n + \max_{d_r \leq q\hat{d}_n} (1 - \delta)d_r(1 - d_r) - c_r d_r + s(q\hat{d}_n - d_r).\end{aligned}$$

The term $(1 - \delta)d_r(1 - d_r) - c_r d_r + s(q\hat{d}_n - d_r)$ is concave unimodal and attains its maximum at $\frac{1 - \delta - c_r - s}{2(1 - \delta)}$. Therefore, $\hat{d}_r = \min\left(\frac{1 - \alpha c}{2}q, \frac{1 - \delta - c_r - s}{2(1 - \delta)}\right)$. Note that $\frac{1 - \alpha c}{2}q > \frac{1 - \delta - c_r - s}{2(1 - \delta)}$ if and only if $c_r + s > (1 - \delta)(1 - q(1 - \alpha c))$, or, written in terms of α , $\alpha < \bar{\alpha} \doteq \frac{(1 - \delta)(q - 1) + c_r + s}{(1 - \delta)qc}$. That is,

$$\hat{d}_r = \begin{cases} \frac{1 - \delta - c_r - s}{2(1 - \delta)} & \text{if } c_r + s \geq (1 - \delta)(1 - q(1 - \alpha c)) \text{ (or } \alpha \leq \bar{\alpha}), \\ \frac{1 - \alpha c}{2}q & \text{otherwise.} \end{cases}$$

■

Proof of Proposition 3.

Let $\Pi(\alpha)$ be the discounted total profit of the firm as a function of α . Then $\Pi(\alpha) = \hat{d}_n(1 - \hat{d}_n) - c\hat{d}_n + \beta \max_{d_r \leq q\hat{d}_n} (1 - \delta)d_r(1 - d_r) - c_r d_r + s(q\hat{d}_n - d_r)$. Now let's calculate $\Pi(\alpha)$ under the two cases we identified.

Case (a): $c_r + s \geq (1 - \delta)(1 - q(1 - \alpha c))$. In this case, $\Pi(\alpha)$ has the form $-c^2\alpha^2/4 + (c^2 - \beta sqc)\alpha/2 + C$, call it $\Pi_a(\alpha)$. This function is concave unimodal and the point that satisfies the first-order condition is $\alpha_a^* = 1 - \frac{\beta sq}{c}$.

Case (b): $c_r + s < (1 - \delta)(1 - q(1 - \alpha c))$. In this case, the profit function, denoted by $\Pi_b(\alpha)$, is again concave unimodal and the point that satisfies the first-order condition is $\alpha_b^* = \frac{c + \beta q(c_r + (1 - \delta)(q - 1))}{c(1 + \beta q^2(1 - \delta))}$.

We can write $\Pi(\alpha) = \Pi_a(\alpha)I_{\{\alpha \leq \bar{\alpha}\}} + \Pi_b(\alpha)I_{\{\alpha > \bar{\alpha}\}}$. We find that $\Pi_a(\bar{\alpha}) = \Pi_b(\bar{\alpha})$ and $\Pi'_a(\bar{\alpha}) = \Pi'_b(\bar{\alpha}) = \frac{c((1 - \delta)(1 - q(1 - c + \beta sq)) - c_r - s)}{2(1 - \delta)q}$. Therefore, $\Pi(\alpha) = \Pi_a(\alpha)I_{\{\alpha \leq \bar{\alpha}\}} + \Pi_b(\alpha)I_{\{\alpha > \bar{\alpha}\}}$ is a continuous function and $\alpha^* = \alpha_a^*$ if $\Pi'(\bar{\alpha}) < 0$, with $\alpha^* = \alpha_b^*$ otherwise. Note that $\Pi'(\bar{\alpha}) < 0 \Leftrightarrow (1 - \delta)(1 - q(1 - c + \beta sq)) - c_r - s < 0 \Leftrightarrow c_r + s > (1 - \delta)(1 - q(1 - c + \beta sq))$. This is the same condition as in the optimal firm-wide analysis. We conclude that the threshold level where d_n becomes a constraint for d_r is the same as under the firm-wide optimal analysis, which is the desired outcome.

If $c_r + s \geq (1 - \delta)(1 - q(1 - c + \beta sq))$, we need to plug in α_a^* in $\hat{d}_n = \frac{1 - \alpha c}{2}$. We find $\hat{d}_n(\alpha_a^*) = \frac{1 - c + \beta sq}{2}$.

If $c_r + s < (1 - \delta)(1 - q(1 - c + \beta sq))$, we need to plug in α_b^* in $\hat{d}_n = \frac{1 - \alpha c}{2}$. We find $\hat{d}_n(\alpha_b^*) = \frac{1 - c + \beta q(1 - \delta - c_r)}{2(1 + \beta q^2(1 - \delta))}$. ■

Proof of Proposition 4. $\frac{\partial C_1}{\partial d_n} = c$ for any d_n . We also need to determine $\frac{\partial C}{\partial d_n}|_{d_n^*}$. At the firm-wide optimal solution, the marginal cost to the firm equals the marginal revenue of the firm: $\frac{\partial C}{\partial d_n} = \frac{\partial R}{\partial d_n} = 1 - 2d_n^*$. In case (i), $d_n^* = (1 - c + \beta sq)/2$ and $1 - 2d_n^* = c - \beta sq$, yielding $\alpha^* = 1 - \frac{\beta sq}{c}$. In case (ii), $d_n^* = \frac{1 - c + \beta q(1 - \delta - c_r)}{2(1 + \beta q^2(1 - \delta))}$ and $1 - 2d_n^* = \frac{c + \beta q(c_r + (1 - \delta)(q - 1))}{1 + \beta q^2(1 - \delta)}$, yielding $\alpha^* = \frac{c + \beta q(c_r + (1 - \delta)(q - 1))}{c(1 + \beta q^2(1 - \delta))}$. ■