Joint Life-Cycle Dynamics of New and Remanufactured Products

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Abstract

Many products considered for remanufacturing are durable that exhibit a well-pronounced product life cycle – they diffuse gradually through the market. The remanufactured product, which is a cheaper substitute for the new product, is often put on the market during the life cycle of the new product and affects its sales dynamics. In this paper, we study the integrated dynamic management of a portfolio of new and remanufactured products that progressively penetrate a potential market over the product life cycle. To this end, we extend the Bass diffusion model in a way that maintains the two essential features of remanufacturing settings: (a) substitution between new and remanufactured products and (b) a constraint on the diffusion of remanufactured products due to the limited supply of used products that can be remanufactured. We identify characteristics of the diffusion paths of new and remanufactured products. Finally, we analyze the impact of levers such as remanufacturability level, capacity profile and reverse channel speed on profitability.

Key words: remanufacturing, product diffusion, closed-loop supply chains

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1 Introduction

How to manage the next-generation product that is introduced to the market before its predecessor has been fully diffused is an important question for many companies, and has been studied extensively. An increasing number of manufacturers are exploring value-added recovery such as remanufacturing, whereby they collect used products and replace worn-out components by new ones. For these firms, the question becomes how to manage the introduction of the remanufactured version of their product while the new version may still be diffusing in the market. In particular, the price evolution and production capacity for both products need to be determined.

Several elements make the joint life-cycle management of new and remanufactured products challenging: First, the remanufactured product reduces (cannibalizes) the sales of the new product when sold on the same market. This is a concern for a wide range of manufacturers such as Xerox, HP, Bosch, and Cummins. Since remanufactured products are often not valued as highly as the new product by consumers, they need to be priced lower than their new counterparts. In addition, prices may evolve over the life cycle of the product. Second, the timing of used product returns constrains sales of remanufactured products. Depending on the product, used products will become available at different points in the new product's life cycle. For example, used photocopiers often return from the market during the life cycle of that photocopier, while used PCs may return later. In the former case, well-functioning components of the photocopier can be reused in a remanufactured photocopier that is sold along with a new photocopier. In the latter case, even though the used PC may contain components that could be remanufactured, there is limited demand for that product. Third, the volume of used product returns constrains sales of remanufactured products. Finally, the diffusion rate of the product in the market, and the repeat purchase potential over the life cycle of the product both impact the value of remanufacturing. For example, with slow-diffusing durable products such as copiers, there is a strong market for remanufactured products, whereas this may not be the case for cell phones.

These issues are major concerns for manufacturers who decide to introduce remanufactured versions of their products. In conjunction with the basic pricing and capacity decisions, three strategic control variables play a role and need to be considered. (1) Product remanufacturability: With a higher level of remanufacturability, a higher fraction of used products can be remanufactured. Increasing remanufacturability can be done through improved product design, durability, data loggers, etc. (2) Design of the reverse channel: The effectiveness of reverse channels in collecting products impacts when returns are available to be remanufactured. Reverse channels can be designed to be more responsive to speed up used product availability (e.g. by means of centralized control). (3) Design of the manufacturing/remanufacturing system. Remanufacturing can be done on the same flexible line as the new product or on a dedicated line. Hewlett Packard remanufactures high-end servers on the same line as new servers, while Michelin retreads used tires on a different production system than new tires.

In this paper, our goal is to answer the following questions:

1. What is the optimal diffusion path for new and remanufactured products? What is the impact of factors such as cannibalization, timing of returns, volume constraints, diffusion rate and repeat purchases?

2. How does the diffusion phenomenon affect the value of remanufacturing, and the remanufacturability level chosen by the manufacturer?

3. How is new and remanufactured product capacity management impacted by diffusion?

4. What is the relative value of flexible capacity, which can be used to both manufacture and remanufacture products, compared to dedicated capacity for each activity?

5. Under what conditions is it more valuable to increase the responsiveness of the reverse channel?

This research is a first attempt to capture life-cycle considerations in the introduction and management of remanufactured products. These are very real considerations in practice that the remanufacturing literature has not addressed to date, and that it needs to incorporate to increase its relevance. Our purpose with this paper is to shed some light on the impact of various trade-offs, and to develop managerial intuition around them.

The remainder of the paper is structured as follows: Our contribution to the literature is outlined in §2. In §3, we introduce our modelling assumptions and formulate the manufacturer's optimization problem. In §4, we characterize product life-cycle dynamics and identify their drivers in uncapacitated and capacitated settings. §5 focuses on the profit impact of varying the strategic control variables we discussed above. In §6, we summarize the implications of our results for the integrated management of product lines with new and remanufactured products. We also discuss the effect of relaxing some of our assumptions. We conclude with directions for future research suggested by our findings.

2 Literature Review

Product life cycle issues in a remanufacturing context have been studied by Geyer et al. (2005), who consider an exogenous (price-independent) product life cycle represented by an isosceles trapezoid. Total demand is given and price-independent, so the focus is on cost minimization. They assume perfect substitution, a constant collection rate, and a constant delay between the sale of a product and its subsequent resale as a remanufactured product. They develop upper and lower bounds on the value of remanufacturing as a function of these parameters and demonstrate that there is a need to coordinate production cost structure, collection rate, product life cycle and component durability to maximize cost savings from remanufacturing. Our paper takes this research further by allowing for price-dependent diffusion, imperfect substitution, and a distribution in the return delay. We determine the optimal time-dependent diffusion trajectory and answer questions about the impact of various managerial levers on profitability.

The diffusion of products on the market has been studied primarily in the marketing literature, the classic example being the Bass diffusion model for one product (Bass 1969). The Bass diffusion model assumes that each customer purchases a product exactly once and uses it during the entire life cycle of the product. Mesak and Berg (1995) analyze the optimal pricing of a single product with replacement sales. The replacement sales at any point in time are assumed to be a constant fraction of the installed base, which is modified by the product price. They find that the total sales (initial and replacement sales) are single peaked. Again for a single product, Kamakura and Balasubramanian (1987) allow a survival function that gives the percentage of products that are expected to survive a number of years after purchase. The authors use this model to estimate the diffusion pattern of different durable.

In order to study the joint diffusion of new and remanufactured products, we allow the product to have a finite *residence time* (duration of one use of the product by a customer) that is shorter in expectation than the *life cycle* (time horizon over which the product stays on the market) of the product. This triggers a remanufacturing opportunity and possible repeat purchases of either new or remanufactured products by customers. We distinguish between the current installed base of products versus cumulative sales, and modify the Bass diffusion model such that the imitation coefficient is a function of the current installed base of the new product. We identify single-peaked and multiple-peaked diffusion patterns depending on product and market characteristics.

The Bass model has been augmented recently by capacity considerations. Ho et al. (2002) and Kumar and Swaminathan (2003) analyze models where diffusion is constrained by capacity. Ho et al. (2002) determine the optimal sales plan for a given capacity level, and also determine the optimal capacity. The profit margin is exogenously given. They find that delaying product launch in order to build initial inventory may be optimal. Kumar and Swaminathan (2003) initially assume that the capacity level is given exogenously and characterize optimal sales plans under backlogging and lost sales scenarios. They then allow capacity to vary over time where the cost of capacity at any given time is convex. In these papers, the focus is on a single product and repeat purchases are not considered. In our two-product setting, in order to sell remanufactured products, used products that can be remanufactured need to be available. This creates an *endogenous* constraint on the sales of remanufactured products, which we include in our model. Production capacity considerations are modelled by assuming linear expansion and contraction costs. In this manner, we analyze the profit impact of capacity costs under sales dynamics introduced by the diffusion phenomenon.

The joint pricing of new and remanufactured products has been studied in Debo et al. (2005) in an infinite-horizon setting with instant diffusion. Groenevelt and Majumder (2001), Ferrer and Swaminathan (2005) and Ferguson and Toktay (2005) model price competition between a manufacturer and an independent remanufacturer in two-period models. These papers do not incorporate life-cycle considerations.

Finally, the impact of delays in the reverse channel has been studied by Blackburn et al. (2004) and Souza et al. (2005) for commercial returns. In these papers, the focus is on increasing the value obtained from rapidly depreciating products by speeding up the time to process them for resale. In our model, obtaining used products earlier allows the manufacturer to sell more remanufactured products during the finite product life cycle, but may increase discounted disposal costs.

To summarize, this paper makes several contributions to the literature. To the remanufacturing literature, it introduces a way to analyze life-cycle dynamics of new and remanufactured products and investigates the impact of various managerial levers (remanufacturability level, capacity structure and reverse channel responsiveness) on profitability. It contributes to the diffusion literature by extending the Bass diffusion model to accommodate repeat purchases, substitution behavior and an endogenous supply constraint.

3 The Model

The manufacturer's goal is to maximize the net present value of introducing a remanufacturable product, calculated over the life cycle of this product, by determining a sequence of prices for the new and remanufactured versions of the product. To model this, we develop a discrete-time, infinite-horizon, discounted profit maximization problem. We introduce assumptions concerning diffusion dynamics, repeat sales, reverse channel, product design, capacity and cost structure in §3.1, and formulate the manufacturer's optimization problem in §3.2.

3.1 Assumptions

The product life cycle consists of gradual diffusion and possible repeat purchases of the product (new or remanufactured). For example, a new cell phone model can have very fast diffusion, but no repeat sales. A tire can have slow diffusion, but many repeat sales. The diffusion rate and the number of repeat purchases determine the length of the product life cycle.

Diffusion of new and remanufactured products.

We denote the *penetrated market size* in period t by M_t. We normalize the total potential market size to 1 so that $M_t \in [0, 1]$. We denote the number of consumers who own a new product in period t by $i_{n,t}$, and call it the *installed base* of new products. With a slight modification of the Bass diffusion model, the potential market size is determined as follows:

$$m_t \doteq M_t - M_{t-1} = (a + b i_{n,t-1})(1 - M_{t-1}) \text{ for } t \geq 0, \tag{1}$$

with $M_{-1} = 0$. In each period, the fraction of new potential consumers (potential first-time purchasers) that is added from the untapped market, $\frac{m_t}{1-M_{t-1}}$, equals a constant, $a > 0$, the 'innovation' coefficient, plus a term proportional to the installed base of new products, $i_{n,t}$, with $b > 0$, the 'imitation' coefficient. Some of these customers will buy the new product, some others will buy the remanufactured product. Note that the word-of-mouth propagation in this diffusion process is driven by the installed base of new products. If there are no repeat purchases, the installed base is equal to cumulative sales, as in the original Bass diffusion model.

Repeat Purchases.

We assume that customers enjoy use of the product for a finite period of time, which we refer to as the "residence time." We allow the residence time to be variable and assume that it has a distribution characterized by $\mathbf{h} = (h_1, h_2, ..., h_L)$ with $\sum_{\tau=1}^{L} h_\tau = 1$: After τ periods of use, a fraction h_τ of new or remanufactured products are returned by the customer. For example, a business may lease copiers for two years, but may decide to extend the lease on some of them for six months. On average, the product stays with the consumer for $\Lambda = \sum_{\tau=1}^{L} \tau h_\tau$ time periods. We consider the residence time distribution as a given characteristic of the product.

At the end of the residence time, customers may choose to make a repeat purchase (new or remanufactured version) of the same product. To model the decreasing likelihood of customers making several repeat purchases, we assume that each customer has a probability ϕ of making a repeat purchase; ϕ is called *the repeat purchase rate*. If $\phi = 0$, then, there are no repeat purchases. If $\phi = 1$, then, all potential customers in any period will become potential customers at most L periods later. Let R_t denote potential repeat customers in period t. Then

$$R_t = \phi \sum_{\tau=1}^{\min(L,t)} h_\tau \left(m_{t-\tau} + R_{t-\tau}\right), \tag{2}$$

where $R_0 = 0$.

Evolution of Sales Volumes.

The prices of new and remanufactured products determine their sales volumes. Let p_N and p_R denote the prices of new and remanufactured products, respectively. Product prices are allowed to be time-dependent; the manufacturer chooses $p_t \doteq (p_{N,t}, p_{R,t})$ in period $t \geq 0$. Let $d_N(p_t)$ and $d_R(p_t)$ denote the corresponding fraction of the potential market that buys new and remanufactured products, respectively, and let n_t and r_t denote the sales volumes of these products in period t. Then

$$n_t = (m_t + R_t)\, d_N(p_t) \quad \text{and} \quad r_t = (m_t + R_t)\, d_R(p_t). \tag{3}$$

To explain this, note that $m_t + R_t$ is the total volume of potential buyers in period t, consisting of new and repeat customers. A fraction d_N and a fraction d_R purchases the new and the remanufactured product, respectively. This is similar to the generalized Bass model (Bass et al. 1994), in which the prices determine the actual sales from potential customers in a multiplicative way. Mesak and Berg (1995) also use the generalized Bass model for single-product diffusion with repeat purchases. Typically, the remanufactured product is an imperfect substitute for the new product. We capture this by imposing $\frac{\partial d_N}{\partial p_R} > 0$, $\frac{\partial d_N}{\partial p_N} < 0$, $\frac{\partial d_R}{\partial p_N} > 0$ and $\frac{\partial d_R}{\partial p_R} < 0$, i.e. when the price of one product increases, its own demand decreases while the demand for the other product increases due to substitution.

Evolution of Installed Base.

In each period, the installed base of new products is increased by new product sales of the current period and decreased by the products sold in previous periods that complete their residence time:

$$i_{n,t+1} = i_{n,t} + n_t - \sum_{\tau=1}^{\min(L,t)} h_\tau n_{t-\tau}. \tag{4}$$

Reverse Channel.

After the residence time **h** during which the customer uses the product, it enters a reverse channel before it becomes available for remanufacturing. The time that the product spends in that channel is Δ. The delay in the reverse channel can be due to not collecting the used product in a timely manner, or, due to having a decentralized, multi-tiered reverse chain. With this model, a fraction h_τ of the customers who buy a product in period t become potential repeat customers in period $t + \tau$, and generate a used product that becomes available to the manufacturer in period $t + \tau + \Delta$.

New Product Design.

The focus of remanufacturing is the subset of components that are usable when the product is returned (e.g. electrical engine of a used power tool, inner casing of a used tire). These components are inspected and the reusable ones are identified for use in remanufacturing. For each such component, what fraction of returns are reusable on average can be identified. We refer to this fraction as the *remanufacturability level*. This level can be influenced by product design, e.g. through appropriate material selection for tires, or through incorporation of a data-logger for power tools (Amezquita et al. 1995, Ishii et al. 1995, Klausner et al. 1998).

While products consist of a number of components, since our focus is not detailed component-level analysis, we define the remanufacturability level of a product as the average fraction of used products that can be remanufactured, denoted by q. We assume that the manufacturer can choose any remanufacturability level $q \in [0, 1]$. The level of remanufacturability is determined at time 0 since it is the initial technology choice that determines this value for all subsequent periods. If the remanufacturability level is set to zero, the product cannot be remanufactured. If the remanufacturability level q is positive, we say the product is *remanufacturable*. We assume that a remanufacturable product can be remanufactured at most once. This is done for tractability, but is also a reasonable assumption for most cases as only few imperfectly substitutable products are truly remanufacturable multiple times.

We assume that the residence time with the customer is not impacted by the remanufacturability level of the product. The two could be linked in practice since higher durability typically implies both higher remanufacturability and longer useful life. Our assumption is reasonable for customer segments who return the product before it breaks down (or before the remanufacturable component breaks down). Most high intensity users fall in this category (copy shops, taxis, car leasing companies), while low-intensity users may keep their product until it is no longer functional. In the former case, which is of main interest for OEMs, the durability of the reusable components and the residence time can safely be assumed to be independent.

As increasing the level of remanufacturability requires more components, or more expensive materials, we model the new product unit costs as an increasing function of q; $c_n(q)$. We denote the unit remanufacturing cost by c_r. We focus on the case where the manufacturing/re-manufacturing technology remains the same over the whole product life cycle, i.e. no new-generation technologies that deliver increased value to the consumers are introduced (see Linton and Bhattacharya, 2001, for a discussion of new technology introduction).

Remanufacturable Product Supply.

In each period, remanufactured product sales are constrained by the availability of returning remanufacturable products. Let $\mathcal{N}_t = \sum_{\tau=1}^{\min(L,t-\Delta)} h_\tau n_{t-\Delta-\tau}$ and $\mathcal{R}_t = \sum_{\tau=1}^{\min(L,t-\Delta)} h_\tau n_{t-\Delta-\tau}$ be the volume of (previously) new and remanufactured products that return from the market at the beginning of period t. As remanufactured products cannot be remanufactured a second time, they need to be disposed when they return. A fraction $1 - q$ of the (previously) new products that return are not remanufacturable and need to be disposed. The remainder are remanufacturable and can either be (i) remanufactured, (ii) disposed or (iii) kept in inventory for usage in future periods. Let f_t be the volume of remanufacturable products that are disposed in period t. Then, the total volume of products that is disposed in period t, denoted by d_t, is composed of (i) returning remanufacturable products the firm chooses to dispose, (ii) returning (previously) new, but non-remanufacturable products and (iii) returning remanufactured products:

$$d_t = f_t + (1 - q)\,\mathcal{N}_t + \mathcal{R}_t. \tag{5}$$

We denote the unit disposal cost by c_d.

Let I_t be the volume of remanufacturable products that are available at the beginning of period t. Then

$$I_{t+1} = I_t - r_t - f_t + q\mathcal{N}_{t+1}; \tag{6}$$

the inventory of remanufacturable products at the beginning of the next period is equal to the inventory at the beginning of the current period, minus the volumes that are remanufactured or disposed, plus the supply of remanufacturable products that become available at the beginning of period $t+1$. In each period, the total volume of remanufactured products and disposed remanufacturable products cannot exceed the available remanufacturable product inventory:

$$r_t + f_t \leq I_t. \tag{7}$$

Note that this model assumes that the manufacturer bears full responsibility to take back all products that he brings on the market. This is consistent with new European legislation. Remanufacturing does not eliminate disposal, but it extends the useful life of new products.

Capacity Costs.

We consider costs associated with maintaining, expanding and contracting production capacity for new products and remanufactured products separately. We denote the cost of operating capacity as c_F per unit of installed capacity; this is independent of the actual production volumes. We assume that the cost of expanding or contracting manufacturing or remanufacturing capacity is proportional to the absolute value of the capacity change. Let $C_t \doteq (C_{n,t}, C_{r,t})$ denote the capacities for new and remanufactured products in period t. We denote the total capacity-related cost in period t by $K\left(C_t, C_{t-1}\right) \doteq K_n \left|C_{n,t} - C_{n,t-1}\right| + K_r \left|C_{r,t} - C_{r,t-1}\right| - c_F \left(C_{r,t} + C_{n,t}\right)$. With these assumptions, the available capacity constrains the sales volumes as follows:

$$n_t \leq C_{n,t} \text{ and } r_t \leq C_{r,t}. \tag{8}$$

3.2 The Optimization Problem

Based on the assumptions outlined above, we can now formulate the monopolist's optimization problem. We define an *implementable diffusion* path $\mathcal{P}_d$ starting with no used remanufacturable product inventory (denoted by $\mathcal{P}_d \in \mathcal{I}(0)$) as a path that is generated by means of an underlying diffusion process: $\mathcal{P}_d \doteq \{(p_t, m_t, R_t, i_{n,t}, C_t), t \geq 0 | I_0 = 0,\ (1),\ (2),\ (3),\ (4),\ (6),\ (5),\ (8)\ \forall t \geq 1,$ and $(7)\ \forall t \geq 0\}$.

With (3), we can write the per period sales profits as $\pi(p_t, q)(m_t + R_t)$ where $\pi(p,q) \doteq d_N(p_t)(p_N - c_n(q)) + d_R(p_t)(p_R - c_r)$. For a given level of remanufacturability, we find the price path that maximizes discounted sales profits minus disposal and capacity adjustment costs:

$$V_\beta(q) \doteq \max_{\mathcal{P}_d \in \mathcal{I}(0)} \sum_{t=0}^{\infty} \alpha^t \left(\pi(p_t, q)(m_t + R_t) - c_d d_t - K(C_t, C_{t-1})\right), \tag{9}$$

with $n_t = r_t = 0$ for $t < 0$ and where α is the discount rate. Due to the complexity of $\mathcal{P}_d$, equation (9) is a non-trivial optimization problem. In all our analysis except that in §4.2.1, we carried out numerical optimization to obtain insights into the solutions of these problems. To facilitate the understanding of the phenomena described, we include figures based on $d_N(p) = 1 - \frac{p_N - p_R}{\delta}$ and $d_R(p) = \frac{p_N - p_R}{\delta} - \frac{p_R}{1-\delta}$. Note that for $\delta \in (0,1)$, $\frac{\partial d_N}{\partial p_R} = \frac{1}{\delta} > 0$, $\frac{\partial d_N}{\partial p_N} = -\frac{1}{\delta} < 0$, $\frac{\partial d_R}{\partial p_R} = -\frac{1}{\delta} - \frac{1}{1-\delta} < 0$ and $\frac{\partial d_R}{\partial p_N} = \frac{1}{\delta} > 0$; new and remanufactured products are imperfect substitutes. δ is referred to as "perceived depreciation." We set the following parameters: $\delta = 0.2$, $\alpha = 0.97$, $c_n(q) = 0.5 - 0.05\ln(1-q)$, $c_r = 0$, $c_F = 0.01$, $c_d = 0$, $K_n = K_r = 0.1$, $\Delta = 10$ and $a = 0.01$. Note that parameter a determines the penetrated market at time $t = 0$. Parameter b determines the impact of the installed base of new products on diffusion. In order to study slow versus fast diffusion without introducing unnecessary dynamics due to initial conditions, we keep a at a low level and vary b. In our model, 'faster' diffusion thus corresponds to a stronger imitation effect. We report experiments with the residence time $\mathbf{h}$ following a symmetric beta distribution over $[1, 20]$ with parameter 2.5. This residence time distribution is illustrated in Figure 1. We report numerical experiments with a time horizon of 200 periods, but show only the first 150 periods, cutting off the end-of-horizon effect. All experiments have been performed using the IPOPT solver, which uses Interior Point optimization techniques and is suited to non-linear constrained optimization problems. The IPOPT solver can be accessed on the web via the NEOS server (http:// www.neos.mcs.anl.gov/neos/).

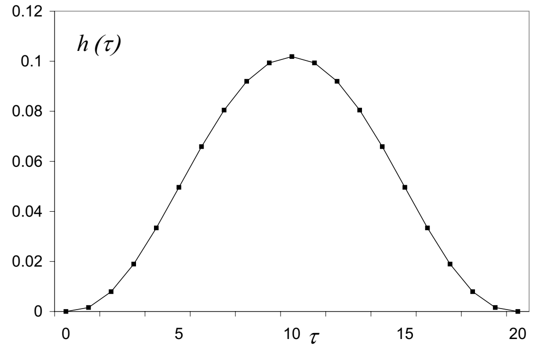

Figure 1: *Residence time distribution,* $h_\tau, \tau = 0, 1, ..., L$*, for* $L = 20$*, and a (discretized) Beta distribution with both parameters* 2.5*. The average residence time is* $\Lambda = 10$*.*

4 Drivers of Diffusion Dynamics

In this section, we discuss diffusion dynamics of new and remanufactured products for a given remanufacturability level q, i.e., we explore properties of the optimal path of sales volumes obtained by solving (9). Since many interacting effects determine the optimal diffusion path, we proceed to understand its drivers in the following steps: We first assume that the capacity adjustment cost is 0. Section 4.1 assumes $q = 0$ (no remanufacturing) and determines the structure of the optimal diffusion path of a single product under repeat purchases. Section 4.2 assumes $q > 0$, thus introducing a remanufactured product to the portfolio. In section 4.2.1, we analytically derive some of the properties of the optimal sales paths under a tractable special case with instantaneous market penetration. Section 4.2.2 investigates joint diffusion dynamics under gradual market penetration. By comparing the results with Sections 4.1 and 4.2.1, we highlight how the diffusion path of the new product changes as a result of remanufacturing and of gradual market penetration. The diffusion path of the remanufactured product is also discussed. Finally, Section 4.2.3 introduces a capacity expansion and contraction cost and investigates its impact on the optimal diffusion path. In Section 5, we investigate the impact of various levers (remanufacturability level, capacity structure and reverse channel responsiveness) on the profitability of the firm. We also investigate how the effectiveness of these levers changes as a function of the diffusion rate and the potential for repeat sales.

4.1 Single Product Diffusion

In this section, we focus on the diffusion of a single product with repeat purchases. First consider a case where customers do not make repeat purchases ($\phi = 0$), which is depicted in the first row of Figure 2 for slow (left panel) and fast (right panel) diffusion. Under the optimal price path, the resulting diffusion curve has a single-peaked structure, like the Bass diffusion model (Bass et al. 1994). This is to be expected since the only difference from the Bass diffusion model is that the installed base is less than cumulative sales due to the finite residence time, so the word-of-mouth effect is weaker. The return stream follows a similar pattern; it is a time-shifted (via the delay in the reverse channel, Δ) and filtered (via the residence time distribution, $\mathbf{h}$) version of the optimal sales path. Due to the spread in residence time, the return peak is lower than the sales peak.

With $\phi > 0$, repeat purchases occur. In this case, we distinguish between first-time sales and repeat sales, which add up to total sales; with $\phi = 0$, first-time sales and total sales are the same. While first-time sales continue to exhibit a single-peaked structure, total sales may oscillate, as demonstrated in the second row of Figure 2. We make the following observation:

Observation 1 *With repeat purchases, we may observe several total sales peaks that dampen over time. This effect is stronger when the time to the first-time sales peak (determined by the diffusion rate) is shorter than the mean residence time.*

To understand this, note that the mean return delay in Figure 2 is $\Lambda = 10$. With fast diffusion, the first-time sales peak takes place before most repeat sales occur and is larger than future total sale volumes. Those customers then return on average 10 periods later, which creates the second sales peak. Due to the spread in the residence time, the second peak is lower and less sharp that the first, and further peaks, if any, quickly dampen. The first-time sales peak for slowly diffusing products is lower than future total sales volumes and occurs later. Therefore, more repeat sales have already occurred by the time this peak is reached so that total sales increase gradually before dropping again due to fewer and fewer repeat purchases.

Observation 2 *The overlap between sales and returns increases as the diffusion rate decreases, regardless of the existence of repeat sales. With repeat sales, there is a higher overlap between sales and returns.*

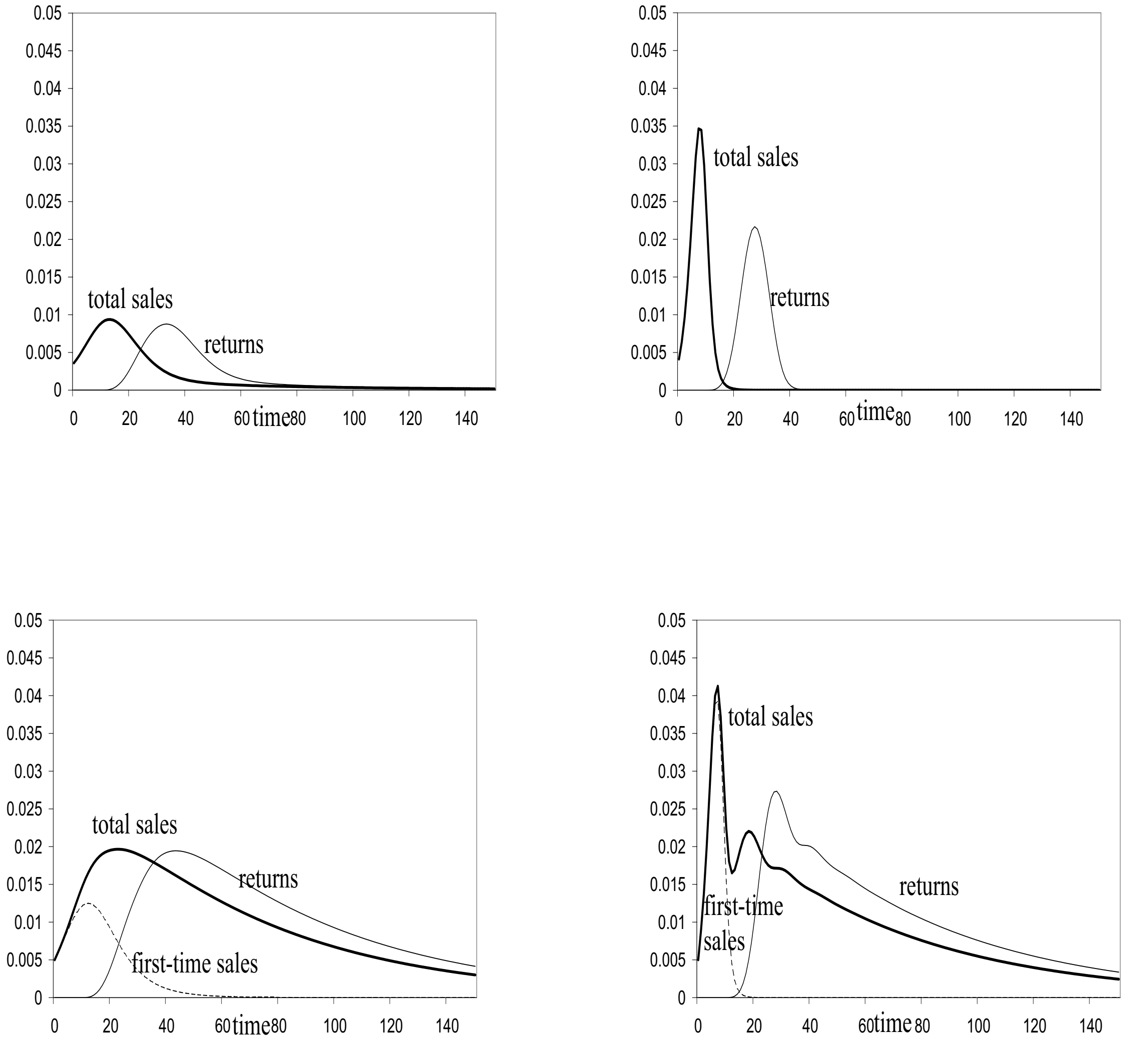

Figure 2: The evolution of first-time sales, total sales and returns with no repeat purchases, $\phi = 0$ (top), and repeat purchases, $\phi = 0.85$ (bottom), with a slow, $b = 0.5$ (left) versus fast, $b = 2$ (right), diffusion rate. When $\phi = 0$, first-time sales are also total sales.

This observation will be of relevance when investigating the impact of different levers on profitability in Section 5.

Finally, we plot discounted profits as a function of the repeat purchase rate and the diffusion rate in Figure 3. As expected, fast-diffusing products generate higher discounted profits since profits are realized earlier, and discounted profits increase with the repeat purchase rate.

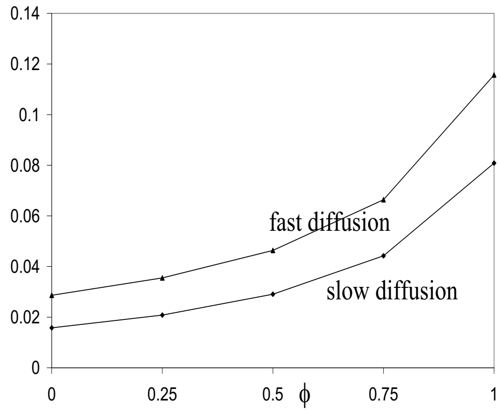

Figure 3: The discounted profits $V_\beta(0)$ as a function of the repeat purchase rate (ϕ) under fast ($b = 2$) and slow ($b = 0.5$) diffusion.

4.2 Joint Diffusion of New and Remanufactured Products

In this section, we consider the joint evolution of the new product and the corresponding remanufactured product, sold by the same firm. To better understand the drivers of joint sales dynamics, we start by considering instantaneous market penetration (§4.2.1), followed by gradual market penetration (§4.2.2).

4.2.1 Instantaneous Market Penetration

The manufacturer's optimization problem (9) is a complex optimization problem. In this subsection, our goal is to analytically solve (9) for a special case to support the observations we make about the general case in §4.2.2 based on numerical experiments. To do this, we focus on the case with a deterministic residence time Λ, instantaneous market penetration ($a \to 0$ and $b \to \infty$) and infinitely repeated sales ($\phi = 1$). We set all disposal and capacity costs to zero. Instantaneous market penetration means all potential consumers have the opportunity to buy the product in the first period. Consumers then make repeat purchases in periods $1+k\Lambda, k \geq 0$. These simplifications allow us to characterize the transient behavior of (9) in closed form. It will be convenient to define the 'aggregate period,' where one aggregate period is equal to Λ periods.

Proposition 1 *Assume a deterministic residence time Λ, instantaneous market penetration ($a \to 0$ and $b \to \infty$) and let $\phi = 1$.*

(a) If only a single product is introduced, the one-period optimal quantity is repeatedly sold in periods $1 + k\Lambda, k \geq 0$.

(b) If the remanufactured product is introduced as well, after some T,

(i) the optimal sales volume of remanufactured products is q times the optimal sales volume of new products in the previous aggregate period. Before period T, not all used remanufacturable products are remanufactured.

(ii) the new product sales path exhibits dampened oscillations determined by the multiplier $z^{\Lambda\tau}$ in aggregate period $\tau \geq T$ with

$$z = -\gamma + \sqrt{\gamma^2 - \frac{1}{\beta}} \in (-1, 0),$$

where $\gamma = \frac{1}{2}\left(q + \frac{1}{\beta(1-\delta)q}\right)$ and $\beta = \alpha^{\Lambda}$.

Proposition 1 shows that repeated sales peaks of a single product are identical while repeated sales peaks of jointly introduced new and remanufactured products fluctuate. To see this, note that $z \in (-1, 0)$. Therefore, the optimal new product sales path displays a fluctuating behavior of decreasing height. We refer to $|z|$ as the amplitude of the oscillation. Since the sales of remanufactured products after period T are exactly equal to the supply of remanufacturable products generated from new product sales of the preceding period, remanufactured product sales exhibit the same pattern, only with smaller height (by a factor q) and shifted by 1 aggregate period. The oscillatory behavior is a result of the negative feedback loop: After being sold, a fraction of the new products enter the market again as remanufactured products and impact the sales of new products in that period. As new and remanufactured products are substitutes, a large volume of new products is accompanied with a small volume of remanufactured products and vice versa.

To summarize, remanufacturing activity causes a fluctuation of the sales peaks, which we note as an observation.

Observation 3 *Under instant diffusion and a deterministic residence time, the sales volume of a single product stays constant. When a remanufactured product is added to the product portfolio, new product sales peaks fluctuate until steady state is reached. Remanufactured product sales peaks also fluctuate, but in counter-phase.*

Our analysis also identifies drivers of the amplitude of the oscillations.

Proposition 2 *With instantaneous product diffusion and a deterministic residence time, the amplitude of the oscillations of the sales peaks ($|z|$) increases as*
(i) the level of remanufacturability increases,
(ii) the perceived depreciation decreases,
(iii) the discount factor decreases,
(iv) the residence time increases.

Proposition 2 identifies key drivers of the fluctuations of the sales peaks. Remember that $z \in (-1, 0)$. Therefore, if $|z|$ increases, the amplitude of the fluctuation increases. Interestingly, a higher level of remanufacturability increases the amplitude of the fluctuations. To understand this, recall that the oscillating behavior is due to the negative feedback loop and substitution. The more remanufacturable the product, the more opportunity there is to sell remanufactured products. This increases the substitution effect, leading to increased sales fluctuations.

Note that when the perceived depreciation, δ, increases, then, the amplitude of the fluctuations decreases. To understand this, note that a higher δ means that consumers place a lower value on the remanufactured product so that the substitution effect is weaker.

Finally, note that a low level of the discount factor, β, or, equivalently, a long (deterministic) residence time Λ is also a driver of fluctuations. To understand this, note that with a very low discount factor, the future profit stream is less important. Therefore, in period 1, as no used remanufacturable products are available, the new product sales are set close to the single-period profit-maximizing *new product* sales, which are higher than steady-state new product sales, due to the lack of substitution. When the discount factor increases, the sales quantities will take future profit streams into account and will be set closer to the steady-state sales quantities. Therefore, the amplitude of the fluctuations will be lower.

4.2.2 Gradual Market Penetration

In this subsection, we investigate the diffusion pattern of new and remanufactured products under gradual diffusion ($0 < b < \infty$, $a \geq 0$), finite repeat purchases ($\phi < 1$) and a general residence time distribution.

Figure 4 plots the evolution of sales under slow versus fast diffusion with no repeat sales versus repeat sales. Compare the first rows of Figures 2 and 4, where we assume each customer makes only one purchase. When the diffusion is slow enough that returns take place while the product is still diffusing in the market (top left panel in Figure 2), remanufactured products can be sold to a portion of the market as their first purchase, as observed in the top left panel in Figure 4. With fast diffusion, this opportunity is limited. We conclude that:

Observation 4 *For slow-diffusing products, remanufacturing can be attractive for a manufacturer even with few repeat sales opportunities.*

In the top left panel of the figure, notice that when remanufactured product sales peak, new product sales are low, and vice versa. This phenomenon is similar to the oscillations identified in Proposition 1 and is driven by the same factors: the substitutability of new and remanufactured products and the time delay in product returns. Taken together, a negative feedback loop is created when new and remanufactured sales overlap, leading to sales oscillations. Now compare the second row of Figures 2 and 4. Recall that with repeat sales, total sales volumes of a single product may oscillate, depending on the speed of diffusion relative to the return delay (as in the bottom right panel of Figure 2). The additional oscillation in new product sales observed in the bottom right panel of Figure 4 is due to the introduction of the remanufactured product. Remanufactured product sales peaks may also exhibit dampened oscillations that are in counter phase with the new product oscillations. Again, this phenomenon is similar to that discussed in Proposition 1 and Observation 3, and the drivers are the same. The effect is modulated by the diffusion rate; the effect is stronger under fast diffusion. Finally, comparing the top and bottom rows of Figure 4 highlights

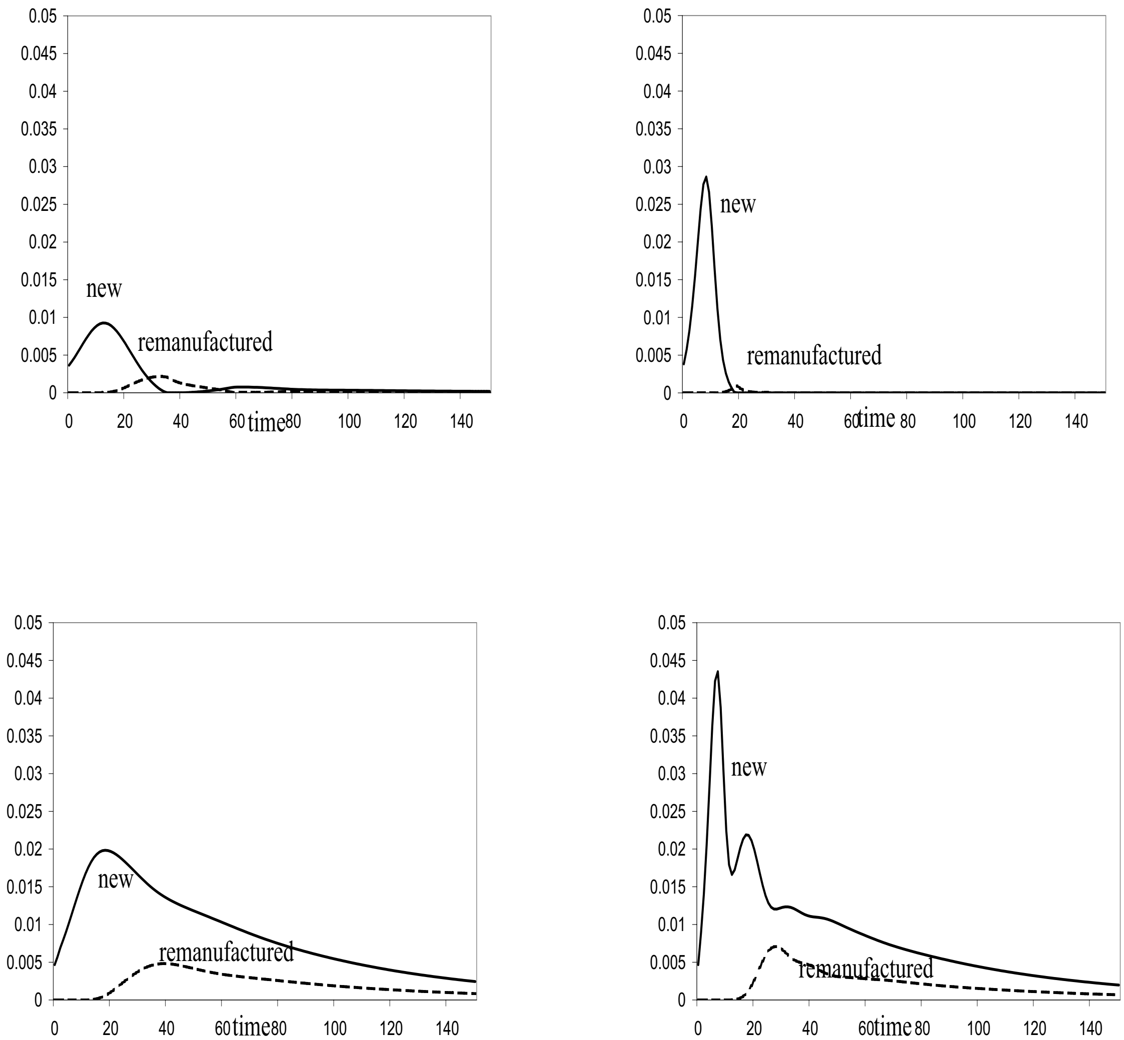

Figure 4: The evolution of new and remanufactured product sales with no repeat purchases, $\phi = 0$ (top), and repeat purchases, $\phi = 0.85$ (bottom), with a slow, $b = 0.5$ (left), versus fast $b = 2$ (right), diffusion rate.

that the opportunity to profitably sell remanufactured products increases with the repeat purchase rate. We summarize this discussion in the following observation.

Observation 5 *Profits from remanufacturing increase with the number of repeat purchases customers make during the life cycle of the product. New product sales may exhibit more oscillations when the remanufactured product is introduced, in counterphase with oscillations in remanufactured product sales.*

4.2.3 The Impact of Capacity Adjustment Costs

We now investigate the impact of capacity adjustment costs. For an example, see the curves in Figure 5 where we make capacity adjustment expensive by taking $K_n = K_r = 0.1$. Comparing this with the uncapacitated benchmark in Figure 4, we note that when it is expensive to adjust capacity, the sales peak is "cut off" and some sales are foregone. While the out-of-phase oscillations we observed in new and remanufactured product sales persist, their magnitude is reduced due to the reduction in the first peak. The optimal capacity investment is higher where the uncapacitated sales peak is higher – under fast diffusion and repeat sales – to capture the revenue potential early on in the product life cycle. We summarize these findings in the following observation:

Observation 6 *With non-zero capacity adjustment costs, the sales peaks on the optimal diffusion path are dampened. The optimal capacity investment level increases with the diffusion rate and the repeat purchase rate.*

5 The Profit Impact of Design Levers

In this section, we study the impact of levers from three different areas (new product design, manufacturing systems design and reverse channel design) on firm profitability. The levers we choose are remanufacturability level, investment into dedicated versus flexible capacity, and reverse channel responsiveness, respectively.

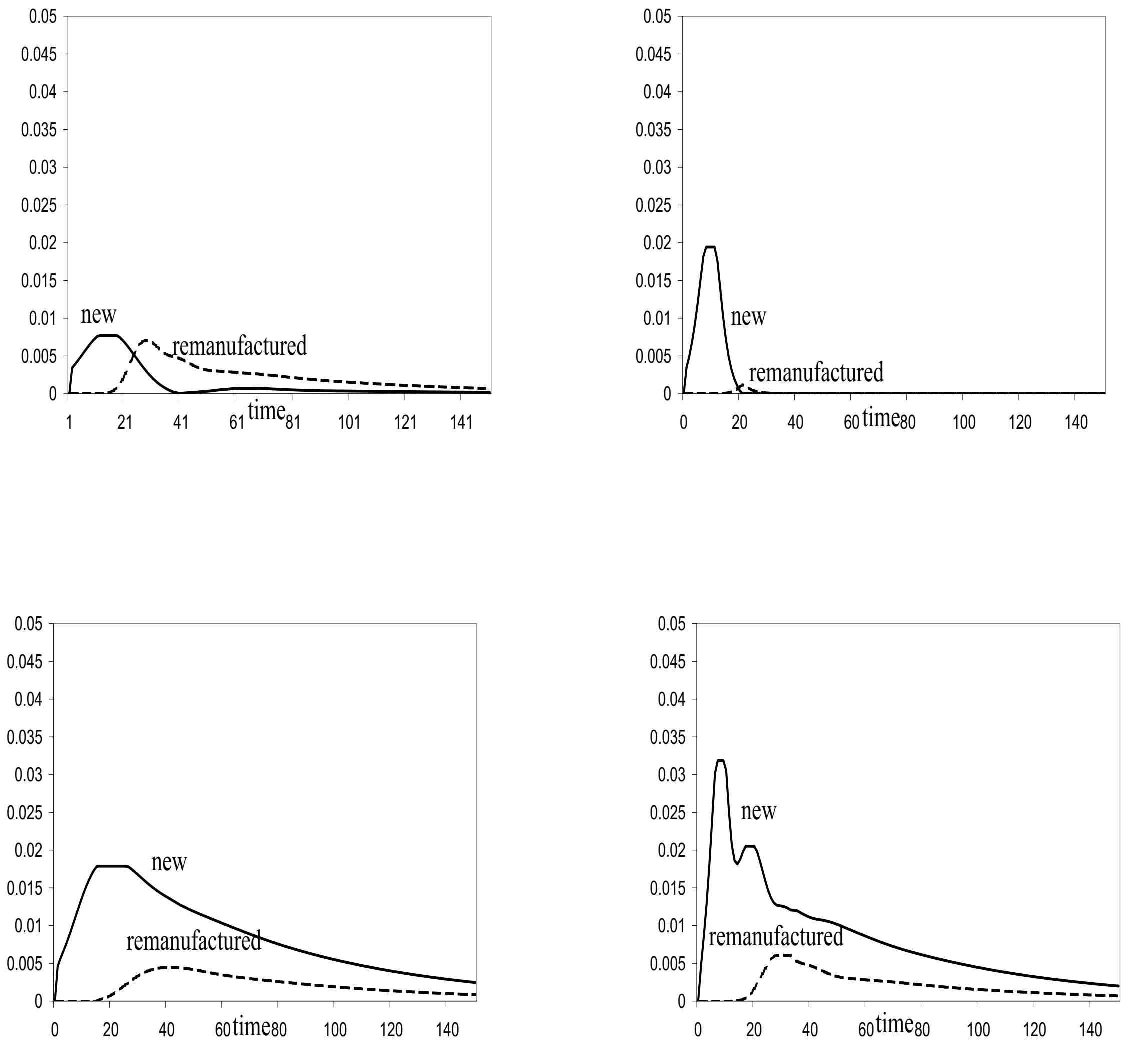

Figure 5: The capacity-constrained evolution of new and remanufactured product sales with no repeat purchases, $\phi = 0$ (top), and repeat purchases, $\phi = 0.85$ (bottom), with a slow, $b = 0.5$ (left), versus fast, $b = 2$ (right) diffusion rate.

5.1 Investing in Remanufacturability

Increasing the remanufacturability of the product does not come for free: Typically, the new product's unit production cost increases. The optimal level of remanufacturability in our model is given by

$$q^* = \operatorname*{argmax}_{q \in [0,1]} V_\beta (q) . \tag{10}$$

The left panel of Figure 6 shows how the optimal level of remanufacturability changes as a function of the repeat purchase rate and the diffusion rate. The optimal remanufacturability level increases with the rate of repeat purchases since there is more opportunity to sell products during the life cycle of the product. For a given repeat purchase rate, the optimal remanufacturability level under slow diffusion is larger than that under fast diffusion: Since there is more opportunity to sell remanufactured products, the firm capitalizes on this by investing in remanufacturability. The difference is larger when the repeat purchase rate is low. This follows from Observation 4: Without repeat sales, the opportunity to sell remanufactured products arises only under slow diffusion.

In the right panel of Figure 6, we plot the profit increase obtained from introducing a remanufactured product and choosing the optimal remanufacturability level q^* compared to introducing a single-use product ($q = 0$). As expected, the gain increases with the repeat purchase rate because total life-cycle sales increase. The increase is sharper under fast diffusion since the remanufacturability level is more sensitive to the repeat purchase rate for fast-diffusing products (as seen in the left panel). At low repeat purchase rates, the gain is larger for slow-diffusing products: As explained in the previous paragraph, the opportunity to sell remanufactured products arises only under slow diffusion, so the firm can increase its profits by investing in remanufacturability. At high repeat purchase rates, the gain is larger for fast-diffusing products. This is driven by discounting: Under fast diffusion, gains from increasing remanufacturability are obtained earlier, increasing the total discounted profit more than with slow diffusion, even if the remanufacturability level chosen is identical.

5.2 Manufacturing System Design and Capacity Flexibility

When setting up a manufacturing/remanufacturing system, different choices can be made concerning the integration of both activities. A separate remanufacturing facility can be set up, or remanufacturing can be performed at the manufacturing facility, where feasible. Our base

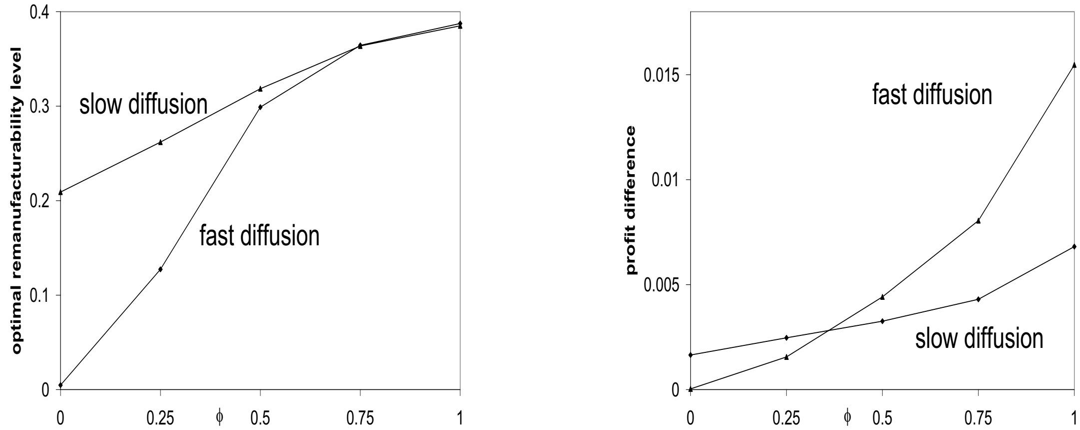

Figure 6: Optimal remanufacturability level (left) and optimal discounted profit differential $V_\beta(q^*) - V_\beta(0)$ (right) as a function of the repeat purchase rate for two diffusion rates; $c_n(q) = 0.5 - 0.05 \ln(1-q)$.

model assumed separate dedicated capacity for the two operations. We now assume that flexible manufacturing/remanufacturing capacity C_t can be used to produce new *and* remanufactured products in period t. With this assumption, the available capacity constrains the sales volumes as $r_t + n_t \leq C_t$. In addition, the cost from changing capacity levels is modelled as $K(C_t, C_{t-1}) \doteq K_f |C_t - C_{t-1}| - c_{F'} C_t$. To separate out the revenue change from the cost impact, we assume equal costs $K_f = K_n = K_r$ and $c_F = c_{F'}$. Since flexibility is more valuable than dedicated resources, the profit differential is positive in Figure 7 as expected. What we wish to underline is the impact of diffusion rate and repeat purchase rate on the differential. Note that both the diffusion rate and the repeat purchase rate contribute to making flexible capacity more valuable. Recall that the uncapacitated diffusion paths exhibit oscillations that are in counterphase, especially with fast diffusion and a high repeat purchase rate, and both diffusion curves are cut off by dedicated capacity constraints (Figure 5). With flexible capacity, only the total volume determines the capacity adjustment costs, and the total volume exhibits less fluctuation (due to individual oscillations being in counterphase), so flexible capacity is less restrictive.

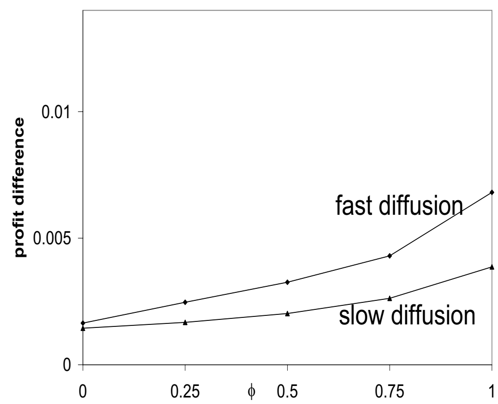

Figure 7: Improvement in discounted profits obtained by switching to flexible capacity as a function of the repeat purchase rate and the diffusion rate.

5.3 Reverse Channel Responsiveness

The delay in the reverse channel, Δ, can be reduced, e.g. by providing incentives to customers for returning their products earlier after the product fails, or by centralizing the control of the reverse chain. In this subsection, we study the impact of having a fast reverse channel on the profits. To separate revenue change from cost impact, we do not assign any cost to speeding up the reverse channel. An example is provided in Figure 8.

When value obtained from remanufacturing dominates the cost of disposal (left panel), reverse channel speed is valuable, and all the more so with fast diffusion and high repeat sales because more sales opportunities exist over the life cycle in this case, and speed is more crucial. When the cost of disposal dominates (right panel), reverse channel speed loses value. This is because disposal needs to take place earlier with a rapid return process, increasing discounted costs over the life cycle of the product. In cases where the manufacturer can inspect or test before taking back its product, reverse channel speed would again be valuable.

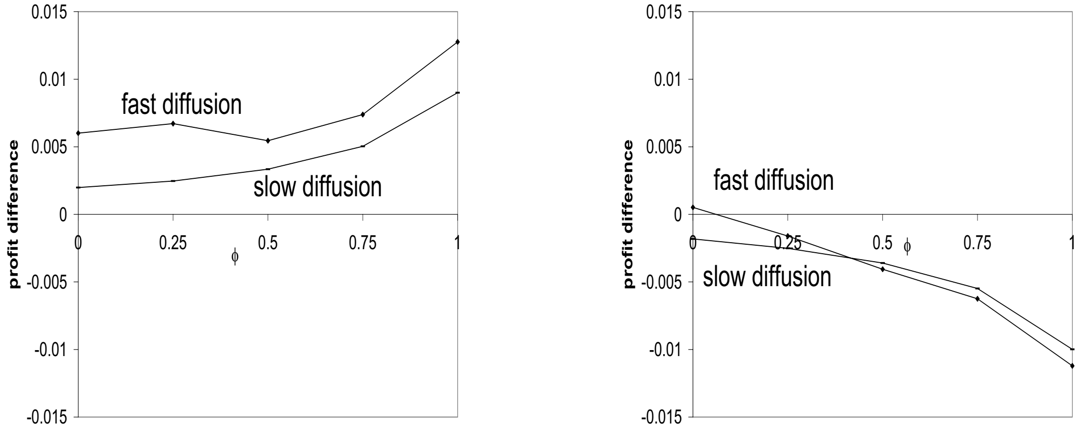

Figure 8: Improvement in discounted profits obtained by having a more responsive reverse channel ($\Delta = 0$), without disposal costs, $c_d = 0$ (left panel), and with disposal costs, $c_d = 0.2$ (right panel).

6 Conclusion, Discussion and Further Research

Summary of the main findings. This paper deals with managing the joint life-cycle dynamics of new and remanufactured versions of the same product. This question is particularly relevant in an environment where an increasing number of firms are remanufacturing their products. The existing diffusion literature primarily focuses on managing a new product introduction while the previous product is still diffusing in the market. Introducing a remanufactured product brings additional considerations: The remanufactured product is an imperfect substitute for the new product and impacts its sales dynamics. It is possible to introduce it very early on in the life cycle of the product. The return volume and remanufacturability level of used products constrain remanufacturing volumes. The reverse channel speed impacts the value that can be obtained from returns. In determining product prices and production capacities over time, the manufacturer needs to take these factors into account. Other questions of interest are what level of remanufacturability to invest in, what capacity structure to choose and whether it is worth increasing the responsiveness of the reverse channel. The answers our analysis gives to these questions are summarized below.

Repeat purchases may create sales fluctuations. With no repeat sales (Bass) or repeat sales that are a fraction of the installed base (Mesak and Berg 1995), the diffusion curve is single-peaked.

For more realism, we allowed repeat sales with a symmetric residence time distribution. If diffusion is slow relative to the mean residence time, a single peak is observed again; this is quite a robust pattern. But if diffusion is fast, there is a high first-time sales peak, followed by sales oscillations that gradually disappear before total sales decline towards the end of the life cycle. A quickly adopted new technology with a long residence time will show this pattern.

Capacity investment should be higher with fast diffusion and a high repeat sales rate. To understand this, we need to remember the uncapacitated diffusion pattern. As we just explained, with fast diffusion, there is an initial high peak before sales stabilize and then decline. With capacity cost, following the uncapacitated optimal path is not cost-effective, thus the optimal solution will limit the high sales peak. On the other hand, to benefit from the opportunity to increase discounted profits by selling more products early on, the capacity investment will be higher than the slow-diffusion case where an initial peak is not observed even without capacity cost. With a high repeat sales rate, the magnitude of the fluctuations is larger, so this effect is enhanced.

Fast diffusion and repeat sales make flexible capacity worthwhile. When new product sales fluctuate, we also observe remanufactured product sales fluctuations, but in counterphase (due to the supply constraint and the time delay). As a result, the total sales fluctuation is dampened relative to the individual sales fluctuations. Flexible capacity takes advantage of this and achieves the same sales with less total capacity investment relative to the dedicated case. Technology and personnel constraints may not allow the same facility to process both new and remanufactured products, but where this is feasible, it can be valuable. For example, HP has recently changed from dedicated to flexible capacity for remanufacturing servers (private communication with Hendrik Brumme, former plant manager of the HP server factory in Germany, January 2006). It is also useful to understand that with slow diffusion or few repeat sales, dedicated capacity is appropriate; an additional investment in flexibility is not warranted.

Slow-diffusing products are the best candidates for remanufacturing. With such products, it is worthwhile to invest in remanufacturability since there is more overlap between the new and remanufactured product life cycles, and therefore more opportunity to sell remanufactured products (e.g. copiers, cars). As the repeat purchase rate increases, the investment into remanufacturability should again increase.

Reverse channel responsiveness is not always valuable. Collecting products brings with

	low repeat purchase rate	high repeat purchase rate
slow diffusion	*Single sales peak* High remanufacturing potential Dedicated capacity most valuable e.g. diesel engine	*Likely single sales peak* Highest remanufacturing potential High remanufacturability most valuable e.g. copiers
fast diffusion	*Some sales fluctuations* Lowest remanufacturing potential Rapid returns most valuable e.g. high-tech cell phones	*Sales fluctuations* Needs high capacity investment Flexible capacity most valuable e.g. high-end servers

Table 1: Policy implications along the diffusion rate and repeat purchase rate dimensions.

it the responsibility to dispose of them if they are not used. We assumed that the firm needs to process all used products and that it takes some time for the product to get back for testing and remanufacturing post use. Under these assumptions, especially if the value of remanufacturing is not very high, bringing in the product faster creates early disposal costs, which reduces discounted profits. If it is possible to only take back reusable products via testing, if there is value depreciation, or if disposal is not costly, then reverse channel responsiveness may be valuable.

Policy implications. This is the first paper that analyzes the joint diffusion of new and remanufactured products; the remanufacturing literature to date has ignored the implications of diffusion and repeat customer purchases. Within our modelling framework, we identify a number of phenomena that advance our understanding of remanufacturing systems. From a managerial perspective, the first step should be to categorize the product with respect to its diffusion rate and the likelihood of repeat purchases. Table 1 outlines the policy implications of our results along these dimensions. While we associated a particular product with each scenario in Table 1 for illustrative purposes, a caveat is in order: Our categorization is at a generic level. Even for the same product, depending on the model, the target customer segment and the sales channel, the product may fall in different categories. For example, in some markets cell phones are adopted much faster than others. The fashion user buys a cell phone every six months, while the basic models are utilized for much longer by the price-conscious segment. Thus, in practice, the product line should be categorized at a more detailed level for these insights to apply.

Discussion of model assumptions. We briefly discuss implications of relaxing some of the model assumptions. We assumed that a product can be remanufactured only once. This assumption was made for tractability. As we discussed earlier, it is nevertheless appropriate in a number of settings. Within our model, we could relax it by assuming that each product has a probability q of being remanufactured after each use. This would increase the supply of remanufacturable products in each period. Consequently, the amplitude of the oscillations would increase (same effect as increasing q) and flexible capacity would be more valuable under fast diffusion and repeat sales. In addition, the value of investing in remanufacturability would decrease since a larger supply is available for a given remanufacturability level.

We assumed that all product disposal costs are incurred by the manufacturer. This assumption is particularly appropriate under extended producer responsibility (e.g. WEEE Directive in Europe) or when the manufacturer needs to collect the product prior to evaluating it. If this is not the case, then the disposal cost for the manufacturer will decrease, and speeding up the reverse channel will be more attractive even for high disposal costs.

We assumed that the likelihood of repeat purchase stays constant. It could be that this likelihood increases (or decreases) over time. In this case, phenomena due to repeat purchases would be enhanced (or reduced), such as the value of investing in remanufacturability, the degree of sales fluctuations, and the value of flexible capacity.

We assumed that the diffusion process is driven by the installed base of new products. Numerical experiments with our model indicate that the results do not differ significantly when the installed base of remanufactured products also contributes to the diffusion process.

We assumed that the remanufacturability level is given by the fraction of products that can be remanufactured. In practice, products have multiple components, only some of which can be remanufactured, and to different degrees. In that sense, using a single remanufacturability level q represents a degree of abstraction. Nevertheless, most products have one or two main components that are the most valuable and that drive the remanufacturing decisions (e.g. steel cord in radial tire, engine in a car), in which case the model is quite appropriate.

Further research. Our approach and findings open several new avenues for research. On the modelling front, remanufacturing research can be enriched to include life-cycle effects, and research on product diffusion can be extended to the remanufacturing domain. For example, the effect of remanufacturing competition has been addressed in a 2-period model (Groenevelt et al. 2001) and in steady state (Debo et al. 2005). The nature of remanufacturing competition at different stages of the product life cycle can be analyzed using our diffusion model, assuming that the pricing decisions are taken by different firms. Another example is diffusion under non-stationarities such as decreasing cost or the introduction of new products (Norton and Bass 1987, Druehl et al. 2004). With remanufacturing, these dynamics will be modified and will impact the firm's technology decisions over time. Our modified diffusion model can be used to incorporate the effect of remanufacturing in these settings. Finally, in the same spirit as the empirical product diffusion literature, there is an opportunity to test some of the implications of product remanufacturing that we identified on the shape of the product life cycle.

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7 Appendix

We reformulate the Propositions in the text in precise mathematical format. Debo et al. (2005) assume that the penetrated market size is already 1 in the first period, and that the residence time is 1 period. Thus by redefining an 'aggregate' period corresponding to Λ periods, we obtain an equivalent model, and can build on their analysis. The discount factor in Debo et al. is β per period. Our model can be cast in their framework by defining the discount factor $\beta \doteq \alpha^{\Lambda}$ corresponding to an aggregate period. Without loss of generality, we set $\Delta = 0$.

Proposition A 1 *For $a \to 0$, $b \to \infty$, $K(C_t, C_{t-1}) = 0$, $h_\Lambda = 1$, $\phi = 1$, and $c_d = 0$, there exists $T \doteq 1 + \tau_0 \Lambda \geq 0$ where τ_0 is integer such that $\tilde{n}_\tau = \tilde{n}_\infty + (\frac{I_T}{q} - \tilde{n}_\infty) z^{1+\tau-\tau_0}$ and $\tilde{r}_{\tau+1} = q\tilde{n}_\tau$ for $\tau \geq \tau_0$, with*

$$z = -\gamma + \sqrt{\gamma^2 - \frac{1}{\beta}} \in (-1, 0),$$

where $\gamma = \frac{1}{2}\left(q + \frac{1}{\beta(1-\delta)q}\right)$. The optimal sales path is given by $n_t^* = \tilde{n}_\tau$ and $r_t^* = \tilde{r}_\tau$ for $t = 1 + \tau\Lambda$ and 0 otherwise. $\tau_0\Lambda$ is such that in periods $t < 1 + \tau_0\Lambda$ not all used remanufacturable products are remanufactured (i.e. $r_t^* < I_t$) and in periods $t \geq 1 + \tau_0\Lambda$, all used remanufacturable products are remanufactured (i.e. $r_t^* = I_t$). $\nu_\infty \doteq (n_\infty, r_\infty)$ satisfies $\frac{\partial \hat{\pi}(\nu_\infty, q)}{\partial n} + q\beta \frac{\partial \hat{\pi}(\nu_\infty, q)}{\partial r} = 0$ and $qn_\infty = r_\infty$.

Proof of Proposition A1. Let $\nu_t \doteq (n_t, r_t)$ be the sales volumes of new and remanufactured products respectively in period t, then, we can rewrite the per period profits as a function of ν by defining $\hat{\pi}(\nu, q) \doteq \pi(\hat{p}(\nu), q)$ with $\hat{p}(\nu) \doteq (1 - n - (1 - \delta) r, (1 - n - r)(1 - \delta))$. As discussed in the text, a problem with $h_\Lambda = 1$, $\Lambda > 1$ and discount factor α can be restated as a problem with $\Lambda = 1$ and discount factor $\beta = \alpha^\Lambda$ by redefining Λ periods as one new period. Debo et al. (2005) formulate and analyze a dynamic program for (9) with $\Lambda = 1$ and discount factor β. We follow their analysis for determining the transient behavior of the system. Please refer to their Technical Appendix for a detailed analysis of the dynamic program. Based on their analysis, we can state that

$$\frac{\partial \pi\left(n_t^*, r_t^*\right)}{\partial n} + \beta q \frac{\partial \pi\left(n_{t+1}^*, r_{t+1}^*\right)}{\partial r} = 0 \ \forall t, \tag{11}$$

where * denotes the optimal path. Also, from Debo et al., we know that $\nu_\infty = (n_\infty, qn_\infty) = \nu^*(I_\infty)$ exists and is unique. Here, $\nu(I)$ is the iteration function: starting with inventory I, the optimal sales quantities are $\nu(I)$. Following Stokey and Lucas (1989, section 6.4), we rewrite the previous expression using Taylor series expansion around ν_∞ for $t \geq T$. We obtain the following second order linear difference equation:

$$\begin{aligned} 0 &= \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial n^2}\left(n_t^* - n_\infty\right) + \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial n \partial r}\left(r_t^* - qn_\infty\right) \\ &+ \beta q \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial r \partial n}\left(n_{t+1}^* - n_\infty\right) + \beta q \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial r^2}\left(r_{t+1}^* - qn_\infty\right). \end{aligned}$$

From Debo et al., we have that $r_t^* = I_t$ and $qn_t^* = r_{t+1}^*$ for $t \geq T$. Let us define $z_t \doteq n_t^* - n_\infty$. Then, the previous equation can be written as:

$$\left(\frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial n^2} + \beta q^2 \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial r^2}\right) z_t + q \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial n \partial r} z_{t-1} + \beta q \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial r \partial n} z_{t+1} = 0, \forall t \geq T \tag{12}$$

with $z_{T-1} = \frac{I_T}{q} - n_\infty$. The solution of (12) is of the form $z_t = a z_a^{t-T} + b z_b^{t-T}$ where z_a and z_b are the roots of the characteristic equation

$$\left(\frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial n^2} + \beta q^2 \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial r^2}\right) z + q \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial n \partial r} + \beta q \frac{\partial^2 \pi\left(n_\infty, qn_\infty\right)}{\partial r \partial n} z^2 = 0. \tag{13}$$

Note that by the definition of $\pi(\nu)$, the Hessian of $\pi(\nu)$ is equal to the Hessian of $R(\nu) \doteq n p_N(\nu) + r p_R(\nu)$. Let us define $R_\infty \doteq R\left(n_\infty, qn_\infty\right)$. Then

$$z_{a,b} = \frac{-\left(\frac{\partial^2 R_\infty}{\partial n^2} + \beta q^2 \frac{\partial^2 R_\infty}{\partial r^2}\right) \pm \sqrt{\left(\frac{\partial^2 R_\infty}{\partial n^2} + \beta q^2 \frac{\partial^2 R_\infty}{\partial r^2}\right)^2 - 4\beta q^2 \left(\frac{\partial^2 R_\infty}{\partial n \partial r}\right)^2}}{2\beta q \frac{\partial^2 R_\infty}{\partial n \partial r}} = -\gamma \pm \sqrt{\gamma^2 - \frac{1}{\beta}}$$

with $\gamma \doteq \frac{1}{2\beta q} \frac{\frac{\partial^2 R_\infty}{\partial n^2} + \beta q^2 \frac{\partial^2 R_\infty}{\partial r^2}}{\frac{\partial^2 R_\infty}{\partial n \partial r}}$. The roots z_a and z_b are real if $\sqrt{\beta} > \frac{1}{\gamma}$, or $\sqrt{\beta}\left(\frac{\partial^2 R_\infty}{\partial n^2} + \beta q^2 \frac{\partial^2 R_\infty}{\partial r^2}\right) < 2\beta q \frac{\partial^2 R_\infty}{\partial n \partial r}$, or $\frac{\partial^2 R_\infty}{\partial n^2} + \beta q^2 \frac{\partial^2 R_\infty}{\partial r^2} - 2\sqrt{\beta} q \frac{\partial^2 R_\infty}{\partial n \partial r} < 0$. The latter inequality can be rewritten as

$\left[1,-\sqrt{\beta}q\right] H \begin{bmatrix} 1 \\ -\sqrt{\beta}q \end{bmatrix} < 0$ with $H = \begin{bmatrix} \frac{\partial^2 R_\infty}{\partial n^2} & \frac{\partial^2 R_\infty}{\partial n \partial r} \\ \frac{\partial^2 R_\infty}{\partial n \partial r} & \frac{\partial^2 R_\infty}{\partial r^2} \end{bmatrix}$. Remember that H is negative definite. Therefore we conclude that the inequality is satisfied and the roots of (13) are real. In addition, both roots are negative with $z_b < z_a$. Note that $z_a z_b = \left(-\gamma + \sqrt{\gamma^2 - \frac{1}{\beta}}\right)\left(-\gamma - \sqrt{\gamma^2 - \frac{1}{\beta}}\right) = \frac{1}{\beta}$, from which it follows that $|z_b| = \frac{1}{\beta |z_a|}$. Hence, if $|z_a| < 1$, then $|z_b| > 1$. Let us now show that $|z_a| < 1$. Since $z_a < 0$, $|z_a| = \gamma - \sqrt{\gamma^2 - \frac{1}{\beta}}$. $|z_a| - 1 < 0 \Leftrightarrow \gamma - 1 < \sqrt{\gamma^2 - \frac{1}{\beta}} \Leftrightarrow q\left(\beta + 1\right) \frac{\partial^2 R_\infty}{\partial n \partial r} > \frac{\partial^2 R_\infty}{\partial n^2} + \beta q^2 \frac{\partial^2 R_\infty}{\partial r^2}$. The left-hand side of the last inequality is linear in β. It is satisfied for $\beta = 0$, as $q \frac{\partial^2 R_\infty}{\partial n \partial r} > \frac{\partial^2 R_\infty}{\partial n^2}$ for the uniform distribution. It is also satisfied for $\beta = 1$, as $0 > \frac{\partial^2 R_\infty}{\partial n^2} + q^2 \frac{\partial^2 R_\infty}{\partial r^2} - 2q \frac{\partial^2 R_\infty}{\partial n \partial r} = \begin{bmatrix} 1 & -q \end{bmatrix} H \begin{bmatrix} 1 \\ -q \end{bmatrix}$, which is negative since H is negative definite. We conclude that $|z_a| < 1$, $|z_b| > 1$ and that $\lim_{t \to \infty} \left|z_b^t\right| = +\infty$. Since $z_t = a z_a^{t-T} + b z_b^{t-T}$ and n_t^* and n_∞ are finite, we must have that $b = 0$. Consequently, $n_t^* = n_\infty + a z^{t-T}$. The constant a is determined using the boundary condition $z_{T-1} = \frac{I_T}{q} - n_\infty$: $a = z_a(\frac{I_T}{q} - n_\infty)$. If $T = 0$, the linearization (12) holds for all $t \geq 0$ with initial condition $I_0 = 0$, which yields $a = -z_a n_\infty$. Therefore, we obtain $n_t^* = n_\infty \left(1 - z_a^{t+1}\right)$ $t \geq 0$.

We will now evaluate γ. Since the marginal consumers $(\theta_{l,\infty}, \theta_{h,\infty})$ corresponding to ν_∞ satisfy $n_\infty = 1 - \theta_{h,\infty}$ and $r_\infty = \theta_{h,\infty} - \theta_{l,\infty}$, we obtain

$$\gamma = \frac{1}{2\beta q}\left(\frac{1}{1-\delta} + \beta q^2\right).$$

■

Proposition A 2 *For $a \to 0$, $b \to \infty$, $K\left(C_t, C_{t-1}\right) = 0$ and $h_\Lambda = 1$, we obtain the following comparative statics:* $\frac{\partial |z|}{\partial q} > 0$, $\frac{\partial |z|}{\partial \delta} < 0$, $\frac{\partial |z|}{\partial \alpha} < 0$ *and* $\frac{\partial |z|}{\partial \Lambda} > 0$.

Proof of Proposition A2. Assuming that $\gamma > \sqrt{\frac{1}{\beta}}$, note that $\frac{\partial z}{\partial \gamma} = -1 + \frac{\gamma}{\sqrt{\gamma^2 - \frac{1}{\beta}}} > 0$ as $\gamma > \sqrt{\gamma^2 - \frac{1}{\beta}}$. Thus, $\frac{\partial z}{\partial q}$ and $\frac{\partial z}{\partial \delta}$ have the same sign as $\frac{\partial \gamma}{\partial q}$ and $\frac{\partial \gamma}{\partial \delta}$, respectively. We obtain that $\frac{\partial \gamma}{\partial \delta} = \frac{1}{2}\frac{1}{\beta q}\frac{1}{(1-\delta)^2} > 0$, so $\frac{\partial z}{\partial \delta} > 0$ and $\frac{\partial |z|}{\partial \delta} < 0$. We can calculate that

$$\frac{\partial \gamma}{\partial q} < 0 \Leftrightarrow \frac{\delta}{1-\delta} > \beta q^2 - 1,$$

which is always satisfied as β and q are less than 1. Therefore, $\frac{\partial |z|}{\partial q} > 0$. Finally, as $\frac{\partial \gamma}{\partial \beta} = -\frac{1}{2}\frac{1}{\beta^2 q}\frac{1}{1-\delta} < 0$ and

$$\begin{aligned}
\frac{dz}{d\beta} &= \frac{\partial z}{\partial \gamma}\frac{\partial \gamma}{\partial \beta} + \frac{\partial z}{\partial \beta} = \left(\left(1 - \frac{\gamma}{\sqrt{\gamma^2 - \frac{1}{\beta}}}\right)\frac{1}{q\left(1-\delta\right)} + \frac{1}{\sqrt{\gamma^2 - \frac{1}{\beta}}}\right)\frac{1}{2\beta^2}, \\
\frac{dz}{d\beta} &> 0 \Leftrightarrow \sqrt{\gamma^2 - \frac{1}{\beta}} - \gamma > -q\left(1-\delta\right) \\
&\Leftrightarrow \frac{1}{\beta q}\left(\frac{1}{1-\delta} + \beta q^2\right) - \frac{1}{\beta q (1-\delta)} > q\left(1-\delta\right) \Leftrightarrow q > q\left(1-\delta\right),
\end{aligned}$$

we obtain that $\frac{\partial z}{\partial \beta} > 0$ and $\frac{\partial |z|}{\partial \beta} < 0$. Using $\beta = \alpha^{\Lambda}$, it follows that $\frac{\partial |z|}{\partial \alpha} < 0$ and $\frac{\partial |z|}{\partial \Lambda} > 0$. ∎