Remanufacturable Product Design and Contracts under Extended Producer Responsibility

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Abstract

Extended Producer Responsibility (EPR) legislation focuses on the life-cycle environmental performance of products. EPR has significant implications for management theory and practice. However, there is very little operations management research that examines the influence of EPR policy parameters on product design, the interactions among these parameters, and strategic managerial responses to combinations of parameters that constitute EPR instruments. We analytically establish optimal product design and pricing decisions by a manufacturer producing and selling a remanufacturable, durable product, in response to various implementations of EPR. We model a single manufacturer supplying a remanufacturable product to a single customer over multiple periods. The customer has a continuing need for the services of the product and optimizes between the costs of product replacement and the costs of operating the product. We model two environmental design attributes of the product that impact costs to both the manufacturer as well as the customer - a one-dimensional "more is better" measure of environmental performance that captures the environmental impact of the product during use, and a measure of product remanufacturability that captures the environmental impact of the product post-use. From an environmental standpoint, we find that higher costs for environmental impacts induce environmentally favorable product designs. However, higher environmental costs result in lower firm profits. Coordinating contracts are known in the literature to lead to higher supply chain profits. We show that from an environmental standpoint as well, coordination in the supply chain is advantageous; design choices in the coordinated case are environmentally superior to those in the uncoordinated case. We present several contracts that can help achieve coordination in the supply chain, and we relate these contracts to installed base management, leasing, quantity discounts, and two-part tariffs. Thus, we demonstrate how the seemingly divergent objectives of environmental benevolence and profitability can be harmonized through coordination.

(*Keywords: Extended Producer Responsibility, Product Design, Supply Chain Coordination, Contracts*)

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1 Introduction

The impact of environmental policies on a company's profitability and success cannot be underestimated. Annual costs for pollution control in the United States have risen by close to six hundred percent since 1972 and now represent approximately two percent of GDP. Environmental considerations daunt managerial decision-making and directly challenge how corporations handle material resources and produce goods and services (Hoffman 2005). Environmental policy-making has recently followed trends of emphasizing preventive measures over end-of-pipe approaches, focusing on environmental performance of products throughout their life-cycles, and favoring market-based approaches over traditional command-and-control approaches. An excellent example of such policy-making is Extended Producer Responsibility (EPR), which focuses on the life-cycle environmental performance of products. There are two related objectives of any EPR policy - shifting responsibility (physically and/or economically, fully or partially) *upstream* toward the manufacturer and away from municipalities, and providing incentives to manufacturers to incorporate environmental considerations into the design of their products (OECD 2005, 2001). Design decisions under EPR thus need to reflect product life-cycle considerations - including manufacturing, post-use return, remanufacturing, and product pricing. However, there is very little operations management research that analytically examines the influence of EPR policy parameters on product design, the interactions among these parameters, and strategic managerial responses to combinations of parameters that constitute EPR instruments.

From a managerial perspective, EPR provides incentives to expand on proactive and preventive upstream measures, consistent with environmental stewardship. Tojo (2004) concludes from a survey-based empirical study that manufacturers of electrical and electronic equipment and cars in Japan and Sweden have implemented design changes (e.g., design for disassembly and reuse) as well as various measures to reduce material consumption and the use of hazardous substances, in response to EPR legislation. Managerial decision-making, however, requires that environmental benevolence be balanced with the goal of profit-maximization. Consider a manufacturer of a diesel engine (Cummins or Volvo), selling to a fleet operator (UPS or DHL). The manufacturer could undertake design measures to improve the environmental performance of the engine during use (e.g., greater energy efficiency that would translate into lower emissions), or make the engine easier to remanufacture (and, hence, minimize the amount of waste generated and disposed) post-use. A profit-maximizing manufacturer would trade-off investments required to implement these measures against the possibility of generating higher revenues or lower costs over the product's economic life. In a durable good context where the manufacturer and the customer interact repeatedly, it is also likely that the parties evaluate alternatives to selling, such as leasing, or solutions-based approaches such as "installed base management". For example, under its "Evergreen Lease", the carpet manufacturer Interface leases a "floor-covering service" for a monthly fee, accepting responsibility for the maintenance and replacement of installed carpeting. This leasing arrangement, together with the use of Interface's completely remanufacturable carpeting material called Solenium, delivers a 35-fold reduction in the flow of materials, significantly lowering material and landfilling costs. The elevator giant Schindler leases "vertical transportation services" in favor of selling elevators because leasing lets it capture the savings from its elevators' lower energy and maintenance costs (see Lovins et al. (1999) for further examples). We address two main research questions in this paper:

1. Do EPR programs provide appropriate incentives to manufacturers to design environmentally beneficial durable products? How do various levers available to regulators in implementing EPR programs compare in inducing environmentally favorable product design?
2. Can alternative, contractual arrangements such as leasing, or solutions-based approaches such as installed base management, facilitate environmentally superior product design while maximizing supply chain profitability?

Specifically, we consider the following scenario. A single manufacturer supplies a remanufacturable, durable product to a single knowledgeable customer.[1] The customer has a continuing need for the services of the product and optimizes between the costs of product replacement and the costs of operating the product. We model two environmental design attributes of the product - a one-dimensional "more is better" measure of environmental performance (such as energy efficiency), and a measure of product remanufacturability, modeled as the fraction (say, by weight) of the product that can be recovered after use. Product deterioration implies that operating costs increase with time. EPR legislation requires the manufacturer to be responsible for the physical take-back of the product.[2] The portion of the product that cannot be recovered and that has to be disposed of, incurs an environmental charge that might be shared between the manufacturer and the customer. In addition, the customer incurs costs for environmentally harmful impacts (such as emissions) during product use. For example, the European Commission recently recommended[3] that airlines be subject to costs for carbon dioxide emissions through the European Union (EU) Greenhouse Gas Emissions Trading Program. The UK Commission for Integrated Transport (CFIT) points out that future transport and climate change policy will involve significant point sources such as truck fleet operators and transportation companies in emissions trading.[4] Another example, apart from the earlier example of a diesel engine, is that of tires. Rate of tire wear is an important performance criterion for tires. During the use phase, tire wear contributes to particulate matter emissions that the US EPA deems as carcinogenic and air-toxic.[5] At the end-of-life stage, tires that are not recovered end up in landfills; many states in the US collect environmental fees on the sale of new tires to fund scrap tire management programs or stockpile cleanup.[6] In the EU, end-of-life tires are covered by a directive banning the disposal of whole tires in landfills.[7]

Thus, design choices by the manufacturer directly affect the production, environmental and product operating costs, and therefore impact the replacement behavior of the customer. This explicit connection between the manufacturer's design choices and demand for the product via optimal product replacements

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[1] The reader is directed to Lund (2003) for a comprehensive account of about 275 firms in the remanufacturing industry. The report covers aspects such as product and process design, sales and marketing, workforce, capital investment, costs, and strategic barriers and opportunities in the industry. According to the report, remanufactured products are tested to original performance specifications, they often carry warranties comparable to that of new products, and they are sold in the market at about 45% to 65% of equivalent new product prices. Frequent and expert buyers who possess substantial ongoing experience in purchasing remanufactured products and in evaluating a remanufactured product's performance objectively are most often found in commercial and industrial markets. These are the markets in which remanufactured products are most common.

[2] Product take-back is an essential feature of many EPR programs; e.g., cars in Sweden, home appliances in Japan, etc. We model product take-back by assuming a sufficiently large cost for waste disposal.

[3] The recommendation will finally be turned into a legislative proposal. *http://www.euractiv.com/Article?tcmuri=tcm:29-144904-16&type=News.*

[4] *http://www.cfit.gov.uk/reports/ccdefra/04.htm.*

[5] *http://www.fhwa.dot.gov/environment/pm.*

[6] *http://www.epa.gov/epaoswer/non-hw/muncpl/tires/basic.htm.*

[7] *http://www.michelin.com/corporate/front/templates/affich.jsp?codeRubrique=92&lang=EN.*

by the customer is novel in our model. Product and process technologies are assumed to be fairly stable and product life-cycles reasonably long so that remanufacturing is viable. EPR instruments available to the regulator are varied, although most instruments fall under the general categories of product recovery targets, environmental performance standards, disposal charges, and charges for environmental impact during product use (see Appendix B for a brief discussion of various EPR instruments employed in different countries).

We examine how such instruments influence upstream design choices (performance and remanufacturability) by the manufacturer, given that the customer makes product replacement decisions optimally. We find that the optimal levels of performance and remanufacturability increase in the cost of waste disposal as well as in the environmental cost during product use. Moreover, we show that the structure of the supply chain in terms of coordination between the manufacturer and the customer, impacts design choices and firm profitability. A fully coordinated supply chain always makes a higher profit than an uncoordinated one; the loss to the supply chain from a lack of coordination can be attributed to a close analog of the classic double marginalization problem in a decentralized bilateral monopoly (Spengler 1950). Interestingly, supply chain coordination is beneficial from an environmental standpoint. We demonstrate that design choices in the coordinated case are environmentally superior to those in the uncoordinated case; we therefore examine several contracts that can help achieve coordination in the supply chain.

The paper is organized as follows. Section 2 includes an overview of the related literature. Section 3 presents the model. Section 4 includes the analysis for the *uncoordinated case* where the manufacturer and customer optimize their profits separately. Section 5 includes the analysis for the *coordinated case* where the manufacturer and the customer are one integrated firm that maximizes the sum total of profits. Section 6 analytically compares profitability and design outcomes between the uncoordinated and coordinated cases. Section 7 discusses coordinating contracts between the manufacturer and the customer. Section 8 includes a numerical illustration of the results. Section 9 concludes with managerial insights and provides directions for future research.

2 Literature

There is a growing stream of literature that studies the interface between operational decisions and the environment in the context of closed-loop supply chains (see Corbett & Kleindorfer (2001a & 2001b) and the papers referenced therein). An excellent account of this research area is provided in Guide & Wassenhove (2003). Significant advances have been made in the treatment of strategic as well as tactical issues such as facility location (e.g., Fleischmann et al. 2001, Jayaraman et al. 1999), product design and technology selection (e.g., Debo et al. 2005), capacity and materials planning (e.g., Souza 2002, Ferrer & Whybark 2001), inventory management (e.g., Keismüller 2003, Mahadevan et al. 2003, Fleischmann et al. 2002, van der Laan et al. 1999), yield management (e.g., Ferrer 2003), coordination and contracting issues related to the collection of used products (e.g., Savaşkan et al. 2004) and competitive issues including cannibalization of new products (e.g., Ferguson & Toktay 2005, Majumder & Groenevelt 2001). We draw from this stream of literature for our model construct. A significant and novel contribution of our work to this stream of literature lies in the explicit incorporation of environmental legislation into managerial decisions related to remanufacturable product design and supply chain coordination.

Several researchers have examined economic and social efficiencies of various policy instruments such as taxes, subsidies, standards, combined taxes/subsidies, and take-back. For example, see Calcott & Walls (2002), Eichner & Pethig (2001), Calcott & Walls (2000), Fullerton & Wu (1998), Palmer & Walls (1997), and Dinan (1993). The typical objective in this stream of research is for the social planner to maximize net social surplus subject to resource constraints, material balance constraints, and production functions. Material input consumption is treated as a surrogate for product design; an environmentally favorable design implies lower material consumption. Customer demand for the manufacturer's product is a function of the quantity produced, product price, income, and quantity of waste disposed. A consistent finding is that a combined tax/subsidy, where there is a tax on the consumption good supply and a subsidy on the demand for material input, can yield the socially optimal product design and quantity of waste. Our focus is instead on the managerial implications of EPR policies that are increasingly being implemented across countries. In addition, we model the manufacturer's design choices more richly, together with the customer's optimal replacement decision.

It turns out that the explicit modeling of customer behavior allows us to link design choices and environmental impacts to the classic Economic Order Quantity (EOQ) model. In our model, the customer incurs operating costs (such as fuel and maintenance) as well as environmental costs (such as emissions charges) during product use. When the customer decides to replace the product, she incurs a replacement cost (set by the manufacturer) and possibly a disposal cost (set by the regulator). The sum of operating and environmental costs above is analogous to the holding cost in the EOQ model, while the sum of replacement and disposal costs is analogous to the setup cost. Thus, while our motivation for considering coordinating supply chain contracts is very different, our results bear resemblance to those in the related OM literature. Monahan (1984), Lee and Rosenblatt (1986), and Weng (1995) discuss optimal quantity discounts offered by a vendor to induce a favorable ordering pattern by a buyer employing an EOQ policy. Under a price-only contract, an independent buyer faces a unit price larger than the manufacturer's marginal cost and therefore chooses an order quantity that is smaller than the supply chain-optimal order size. Quantity discounts can then be used to encourage the customer to raise her order size to the supply chain-optimal order quantity. In our setting, the customer in the uncoordinated case chooses a replacement interval that is larger than the supply chain-optimal replacement interval. The difference is interesting - the unit price charged by the manufacturer affects holding costs in the EOQ model, whereas it affects the "setup cost" in our setting. Moreover, the price that the manufacturer can charge for the product decreases in the operating and environmental costs incurred by the customer which, in turn, depend upon product design choices by the manufacturer. Additional benefits to the supply chain from more frequent replacements by the customer lie in the manufacturer's incentive to invest in environmentally favorable product design to reduce its share of "setup" and "holding" costs. Prior research abstracts from product design considerations that directly impact costs to both the manufacturer as well as the customer; the factors driving product replacements (or, equivalently, customer demand over a given period of time) are more intricate in our model. We show that coordination of the customer's product replacements leads not only to higher supply chain profit but also to environmentally superior product design choices. The coordinating contracts that we recommend in Section 7 reflect the general concepts of leasing and installed base management applied to remanufacturable products (see Bhattacharya et al. 2005). It is appealing to note the analogies between these contracts and the contracts discussed in the aforementioned literature.

3 The Model

A single manufacturer supplies a remanufacturable, durable product to a single customer. The customer has a continuing need for the services of the product. The manufacturer faces the decision of choosing optimal levels of two attributes of the product - one which determines the environmental performance of the product during product use, and the other which determines the product's environmental impact post customer-use. We model a one-dimensional "more is better" measure of environmental performance during product use q, analogous to the modeling of product performance or quality in Chen (2001), Kornish (2001), Kim & Chhajed (2000), and Moorthy & Png (1992). The product's remanufacturability is determined by the second attribute θ. Debo et al. (2005) and Fleischmann et al. (2001) model the choice of remanufacturability by the manufacturer as the fraction of products that can be remanufactured after use. In an analogous manner, and similar to Fullerton & Wu (1998), we model $\theta \in [0, 1]$ as the fraction of the product (say, by weight) that can be re-used, recovered, or remanufactured after use. As in Debo et al. (2005) and Calcott & Walls (2002), θ determines the cost of production; the cost of production is bounded above by the cost of manufacturing a new product, and decreases in θ. In order to link the customer's replacement decision with the manufacturer's optimal choices of performance and remanufacturability, we utilize an *economic life* model to characterize the customer's replacement decision. The economic life model involves determining the optimum point in time to replace equipment (Dean 1961). We use the replacement model suggested by Clapham (1957), which determines the economic life of equipment by minimizing the average sum of capital and (increasing) operating costs per period. Let $c(q, \theta)$ denote the cost of producing the product, $k(q, \theta)$ denote the design cost to the manufacturer of providing q and θ, $u(q)$ denote the gradient of the product's operating costs per period, $e(q)$ denote the gradient of the environmental costs per period during product operation (e.g., monetary charges for emissions), and $w(\theta)$ denote the cost of waste disposal (e.g., landfilling costs). Denote α and $\beta = 1 - \alpha$ as the fractions of the cost of waste disposal borne by the customer and manufacturer respectively. Assuming uniform product usage over time, let τ denote the optimal time interval between successive product replacements by the customer. We make the following assumptions.

A1 $c_q > 0,\ c_{qq} > 0,\ c_\theta < 0,\ c_{\theta\theta} = 0,\ c_{q\theta} < 0$

A2 $k_q > 0,\ k_{qq} > 0,\ k_\theta > 0,\ k_{\theta\theta} > 0,\ k_{q\theta} = 0$

A3 $u_q < 0,\ u_{qq} > 0$

A4 $e_q < 0,\ e_{qq} > 0$

A5 $w_\theta < 0,\ w_{\theta\theta} = 0$

A6 $w(\theta = 0) > c(q, \theta = 0)$

Assumption A1 implies that the cost of producing the product decreases linearly with remanufacturability θ and increases convexly in performance q.[8] Assumption A2 implies that the initial design cost to the manufacturer increases convexly and separably in q and θ. Assumptions A1 and A2 are consistent with those in Debo et al. (2005), Kim & Chhajed (2002), Kouvelis & Mukhopadhyay (1995), and Moorthy & Png (1992). Assumption A3 implies that a higher performance level translates convexly into a lower gradient of product operating costs per period. Assumption A4 implies that product performance (such as energy efficiency) decreases the cost of environmental impact during product use in a convex manner. Note that $e(q)$ represents a linear degradation of the product's environmental performance with respect to time and, hence, a linear increase in environmental costs during product use with respect to time. Assumption A5 implies that the cost of waste disposal (such as landfilling costs) at the end of the product's economic life decreases linearly in the amount of waste disposed of (see EPA 1998).[9] Assumption A6 sufficiently implies that the cost of product disposal is always greater than the cost of production so that the manufacturer always has an incentive to take back the product instead of disposing it of completely.

EPR instruments differ in the magnitudes of e, w and α, and possibly in the specification of lower bounds on q and θ in the form of design standards q_s and θ_s, respectively. For expositional convenience and without loss of generality, we set q_s and $\theta_s = 0$. We explore the incentives for environmentally favorable product design by the manufacturer for different implementations of EPR under two scenarios - when the customer and manufacturer are separate entities and optimize their profits separately (uncoordinated case), and when the customer and manufacturer are one integrated entity that optimizes total profit (coordinated case). The manufacturer's design and price decisions impact the customer's optimal replacement policy. Performance and remanufacturability choices by the manufacturer thus depend upon the trade-offs in terms of the relative frequencies and magnitudes of revenue or profit earning instances corresponding to product replacements by the customer.[10] The sequence of decisions is as follows. In a given EPR scenario, the manufacturer first chooses q and θ jointly. He then chooses the price $r > c$ to be charged to the customer for product replacements. The customer buys the product at price r if she makes her reservation profit from employing an optimal replacement policy, given q, θ, and r. We proceed by backward induction according to the sequence of decisions in order to arrive at the optimal values of the decision variables.

4 Uncoordinated Case

In this section, we explore optimal design and price choices by the manufacturer when the manufacturer and customer share common knowledge about q and θ but optimize their profits separately. Using backward induction, we first solve for the customer's optimal replacement policy as a function of the manufacturer's design and price choices. The optimal replacement policy is then fed into the manufacturer's optimal price decision. Finally, we deduce the manufacturer's optimal design choices of performance and remanufac-

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[8] Incorporating variable costs independent of θ and q does not change the analysis or affect the nature of results. An implicit assumption is that replacements are readily available to the customer. Since we model a single product being remanufactured and sold in each replacement instance, incorporating fixed costs into the model will not change the analysis or the nature of results.

[9] Information on landfilling in the US is available at *http://www.epa.gov/epaoswer/non-hw/muncpl/disposal.htm.*

[10] We assume common knowledge about costs and design choices of q and θ.

turability, given the optimal price to be charged to the customer and the customer's optimal replacement policy.

4.1 Customer's Problem

In each period, the customer earns a revenue ϕ and incurs operating as well as an environmental costs by using the product. Each time the product is replaced, the customer pays r (i.e., the price set by the manufacturer) and a portion of the product disposal fees (set by the regulator). The customer chooses a replacement interval with the objective of maximizing average profit per period, *a la* Clapham (1957).[11] The customer's problem can be written as:

$$\begin{aligned} \max_t \Pi_C &= \phi - \frac{r}{t} - \frac{1}{t}\int_0^t (u+e)x\ dx - \frac{\alpha w}{t} \qquad (1) \\ &= \phi - \frac{(r+\alpha w)}{t} - \frac{(u+e)t}{2} \qquad (2) \end{aligned}$$

where t is the replacement interval. The optimal time interval between replacements is the age which maximizes Π_C, and is given by

$$t^* := \tau = \sqrt{\frac{2(r+\alpha w)}{u+e}} \qquad (3)$$

The above expression is analogous to the familiar EOQ formula for choosing the optimal order quantity, given the fixed cost of ordering, the unit cost of the product being ordered, and the inventory holding cost rate. In our model, the sum of the price of the product and the customer's share of the cost of waste disposal is analogous to the fixed ordering cost in the EOQ model. The sum of the operating and environmental costs per period is analogous to the holding cost rate in the EOQ model.[12] Observation 1 lists properties of the customer's optimal replacement interval τ with respect to the replacement cost r, the cost of waste disposal w, the customer's share of the waste disposal cost α, the operating cost gradient u, and the environmental cost gradient e.

Observation 1

i. $\frac{\partial \tau}{\partial r} > 0$, $\frac{\partial^2 \tau}{\partial r^2} < 0$; $\frac{\partial \tau}{\partial w} > 0$, $\frac{\partial^2 \tau}{\partial w^2} < 0$; $\frac{\partial \tau}{\partial \alpha} > 0$, $\frac{\partial^2 \tau}{\partial \alpha^2} < 0$.

ii. $\frac{\partial \tau}{\partial u} < 0$, $\frac{\partial^2 \tau}{\partial u^2} > 0$; $\frac{\partial \tau}{\partial e} < 0$, $\frac{\partial^2 \tau}{\partial e^2} > 0$.

Proof: See Appendix A.

The above properties are intuitive. Observation 1.i implies that the optimal time interval τ between replacements increases in a concave manner with the customer's replacement cost r, the cost of waste disposal w, and the customer's share of the cost of waste disposal α. A higher replacement cost translates into less frequent replacements; the frequency of replacements can be expected to be decreasingly influenced by increasing replacement costs. A larger cost of waste disposal delays replacement and this effect can

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[11] The customer's replacement behavior is similar in the discounted-profit model by Bellman (1955) that lacks tractability. The analysis of customer replacement behavior for the Bellman model is available from the authors.

[12] The EOQ formula for the optimal order quantity is given by $Q^* = \sqrt{\frac{2DS}{H}}$ where D is the demand rate, S is the fixed ordering cost, and H is the holding cost rate per unit (Chase et al. 2005). The optimal order interval is, therefore, $\frac{Q^*}{D} = \sqrt{\frac{2S}{DH}}$. In our model, one unit of the product is ordered in each replacement instance.

be expected to diminish as the cost of waste disposal increases. Observation 1.ii implies that τ is convex decreasing in the operating cost and environmental cost gradients. Larger operating and environmental cost gradients with respect to time result in more frequent replacements, and the frequency of replacements can be expected to be decreasingly influenced by increasing cost gradients. Substituting τ from (3) for t in (2) we have

$$\Pi_C = \phi - \sqrt{2(r+\alpha w)(u+e)} \tag{4}$$

Without loss of generality, the participation constraint for the customer is $\Pi_C \geq 0$, or

$$\phi - \sqrt{2(r+\alpha w)(u+e)} \geq 0 \tag{5}$$

Having analyzed the customer's optimal replacement policy as a function of the manufacturer's decisions and the parameters of the model, we proceed to solve the manufacturer's problem.

4.2 Manufacturer's Problem

For consistency with the customer's objective and for analytical convenience, we assume that the manufacturer's objective is to maximize average profit per period.[13] Let T denote the planning horizon. For instance, the planning horizon could be the duration of time for which a particular product version or model is offered. The optimization problem for the manufacturer subject to the customer's participation constraint in (5) is

$$\max_{(q,\theta),r} \Pi_M = \frac{(r-c)}{\tau} - \frac{\beta w}{\tau} - \frac{k}{T} \tag{6}$$

Manufacturer's Price Decision:

We work by backward induction and evaluate the manufacturer's optimal price decision given design choices of q and θ. The customer's optimal replacement interval can be plugged into (6). Proposition 1 provides a useful result.

Proposition 1 *Given design choices of q and θ, the manufacturer's profit increases concavely in the price $r > c$ charged to the customer.*

Proof: See Appendix A.

The above result, though straightforward, is not obvious because it is possible for the manufacturer to trade away the magnitude of revenue in each replacement instance for a higher frequency of product replacements. Corollary 1 follows directly from Proposition 1.

Corollary 1 *It is optimal for the manufacturer to price the product at $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$.*

The manufacturer's participation requires

$$\begin{aligned} r^* &\geq c + \beta w && (7)\\ \text{i.e., } \frac{\phi^2}{2(u+e)} - \alpha w &\geq c + \beta w \\ \text{i.e., } \phi^2 &\geq 2(c+w)(u+e) && (8) \end{aligned}$$

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[13] The problem becomes analytically intractable when discounting is considered. Numerically, it can be shown that the qualitative nature of results remains unchanged.

Thus, we have solved for the manufacturer's optimal price and the customer's optimal replacement policy as a function of q, θ, and the parameters of the model. We now proceed to the first stage in the sequence of decisions where the manufacturer chooses optimal levels of performance and remanufacturability jointly.

Manufacturer's Design Decision:
We have $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ and $\tau = \frac{\phi}{u+e}$. The behavior of the manufacturer's profit function with respect to its design choices of q and θ can be dissected with the help of the following observations.

$$\frac{\partial \Pi_M}{\partial q} = \frac{1}{\tau}\frac{\partial r^*}{\partial q} + \frac{-1}{\tau}\frac{\partial c}{\partial q} + \frac{-(r^* - c - \beta w)}{\tau^2}\frac{\partial \tau}{\partial q} + \frac{-1}{T}\frac{\partial k}{\partial q} \tag{9}$$

$$\frac{\partial \Pi_M}{\partial \theta} = \frac{1}{\tau}\frac{\partial r^*}{\partial \theta} + \frac{-1}{\tau}\frac{\partial c}{\partial \theta} + \frac{-\beta}{\tau}\frac{\partial w}{\partial \theta} + \frac{-(r^* - c - \beta w)}{\tau^2}\frac{\partial \tau}{\partial \theta} + \frac{-1}{T}\frac{\partial k}{\partial \theta} \tag{10}$$

We note the following:

i. $\frac{\partial c}{\partial q} > 0$; $\frac{\partial k}{\partial q} > 0$; $\frac{\partial u}{\partial q} < 0$; $\frac{\partial e}{\partial q} < 0$.

ii. $\frac{\partial c}{\partial \theta} < 0$; $\frac{\partial k}{\partial \theta} > 0$; $\frac{\partial w}{\partial \theta} < 0$.

iii. $\frac{\partial r^*}{\partial q} = -\frac{\phi^2}{2\sqrt{(u+e)}}\left[\frac{\partial u}{\partial q} + \frac{\partial e}{\partial q}\right] > 0$.

iv. $\frac{\partial \tau}{\partial q} = -\frac{\phi}{\sqrt{(u+e)}}\left[\frac{\partial u}{\partial q} + \frac{\partial e}{\partial q}\right] > 0$.

v. $\frac{\partial r^*}{\partial \theta} = -\alpha\frac{\partial w}{\partial \theta} \geq 0$ for $\alpha \geq 0$.

vi. $\frac{\partial \tau}{\partial \theta} = 0$.

The first term on the right hand side of (9) is > 0 since $\frac{\partial r^*}{\partial q} > 0$. This term represents the increase in profit due to a higher price which the manufacturer can charge for a superior performance product. The second term on the right hand side of (9) is < 0 since $\frac{\partial c}{\partial q} > 0$. This term represents the decrease in profit due to a higher unit cost for a superior performance product. The third term on the right hand side of (9) is < 0 since $\frac{\partial \tau}{\partial q} > 0$. This term represents the decrease in profit due to less frequent replacements by the customer if the product has superior performance and, consequently, lower operating and environmental costs. The fourth term on the right hand side of (9) is < 0 since $\frac{\partial k}{\partial q} > 0$. This term represents the decrease in profit due to higher design costs for a superior performance product. The first term on the right hand side of (10) is ≥ 0 since $\frac{\partial r^*}{\partial \theta} > 0$ if $\alpha > 0$, and $\frac{\partial r^*}{\partial \theta} = 0$ if $\alpha = 0$. When the customer incurs a non-zero fraction of the cost of waste disposal, a greater level of remanufacturability reduces this cost and enables the customer to afford a higher product price, thereby contributing positively to manufacturer profit. The second term on the right hand side of (10) is > 0 since $\frac{\partial c}{\partial \theta} < 0$. This term represents the increase in profit due to lower production cost for a product with greater remanufacturability. The third term on the right hand side of (10) is > 0 since $\frac{\partial w}{\partial \theta} < 0$. This term represents the increase in profit due to a decrease in waste disposal costs from a greater level of remanufacturability. The fourth term on the right hand side of (10) is $= 0$ since $\frac{\partial \tau}{\partial \theta} = 0$. The optimal replacement interval is unaffected by the remanufacturability level.

This seems surprising because the expression for the optimal replacement interval in (3) includes a term in w which depends on θ. However, this term vanishes when the optimal price r^* is substituted for r in (3). Note that $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$. Therefore, $r^* + \alpha w$ is independent of θ. In other words, any benefit of lower waste disposal costs[14] to the customer from greater product remanufacturability is extracted back by the manufacturer via a higher product price with the net result being that the optimal replacement interval is independent of product remanufacturability. The fifth term on the right hand side of (10) is < 0 since $\frac{\partial k}{\partial \theta} > 0$. This term represents the decrease in profit due to higher design costs for a product with greater remanufacturability.

While we are able to describe how various factors individually influence the manufacturer's profit, we require specific functional forms for k, c, u, e, and w in order to facilitate exposition and provide holistic insights into the problem. We assume that the fixed cost of design is separable in q and θ and is of the form $k := k_1 q^2 + k_2 \theta^2$, where $k_1,\ k_2 > 0$. We assume that the cost of production is $c := c_0 q^2 (1-\theta)$. Thus, the cost of production decreases linearly in the remanufacturability of the product and is bounded above by the cost $c_0 q^2$ of manufacturing a new product. We assume $u := \frac{u_0}{q}$. Thus, u decreases convexly in q but does not depend on θ. Other possibilities exist for the customer's operating cost gradient u and the customer's revenue ϕ. u could possibly depend on θ if there is a perceived increase in operating inferiority from using a remanufactured product which feeds into the customer's optimal replacement decision. Also, we assume that ϕ is constant in each period, independent of q. The case when the customer's revenue is independent of the performance level q is apt in situations where q impacts operating costs but not revenue. An example is when q represents energy efficiency. Examples where ϕ would increase in q include situations where q impacts equipment uptime or a feature like torque. A higher value of q would then imply that the customer can earn larger revenues for the same duration of product usage. Combinations of different functional assumptions yield various scenarios. However, we focus on the assumed functional forms. We assume w to be of the form $w_0(1-\theta)$, implying that the cost of waste disposal decreases linearly in the amount of waste disposed. Finally, we assume $e := \frac{e_0}{q}$; i.e., that the cost of environmental impact during product use decreases convexly in performance. With these functional forms, we now look at optimal design choices by the manufacturer. In the first stage of the sequence of decisions, the manufacturer optimizes its profit jointly with respect to q and θ, given the optimal price to be charged for the product and the customer's optimal replacement policy. Substituting $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ (from Corollary 1) and $\tau = \frac{\phi}{u+e}$ into the manufacturer's profit function in (6), we have

$$\Pi_M = \frac{\phi^2 - 2(c+w)(u+e)}{2\phi} - \frac{k}{T} \tag{11}$$

Proposition 2 provides properties of the manufacturer's profit function with respect to q, θ, and the parameters of the model.

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[14] Recall that the customer's share of the waste disposal cost is αw.

Proposition 2

i. $\exists\ \bar{k}_2$ such that $\forall\ k_2 > \bar{k}_2$, the manufacturer's profit function is jointly concave in q and θ.[15] (q^*, θ^*) uniquely maximizes Π_M, where $q^* = \left\{q : \frac{\partial \Pi_M}{\partial q} = 0\right\}$ and $\theta^* = \left\{\theta : \frac{\partial \Pi_M}{\partial \theta} = 0\right\}$.[16]

ii. $\frac{\partial \Pi_M}{\partial w_0} < 0$; $\frac{\partial^2 \Pi_M}{\partial w_0^2} = 0$.

iii. $\frac{\partial \Pi_M}{\partial e_0} < 0$; $\frac{\partial^2 \Pi_M}{\partial e_0^2} = 0$.

iv. $\frac{\partial \Pi_M}{\partial \phi} > 0$; $\frac{\partial^2 \Pi_M}{\partial \phi^2} < 0$.

Proof: See Appendix A.

The optimal values of q and θ can be derived from the first order conditions $\frac{\partial \Pi_M}{\partial q} = 0$ and $\frac{\partial \Pi_M}{\partial \theta} = 0$. Thus, we have

$$q^* = \frac{\sigma^2 - c_0\sigma(u_0+e_0)(1-\theta)T + c_0^2(u_0+e_0)^2(1-\theta)^2T^2}{6k_1\phi\sigma} \tag{12}$$

where $\sigma = \Big[(u_0+e_0)(1-\theta)T[54k_1^2w_0\phi^2 - c_0^3(u_0+e_0)^2(1-\theta)^2T^2 - 6k_1\phi(81k_1^2w_0^2\phi^2 - c_0^3w_0(u_0+e_0)^2(1-\theta)^2T^2)^{1/2}]\Big]^{1/3}$. Also,

$$\theta^* = \frac{(u_0+e_0)(c_0q^2+w_0)T}{2k_2\phi q} \tag{13}$$

Note that $\sigma > 0$ and $q^* \geq 0$. Also, $0 < \theta^* < 1$ for $k_2 > \bar{k}_2$. Thus, the only assumption needed for the manufacturer's profit to be jointly concave in q and θ is that the coefficient of the design cost of remanufacturability be sufficiently large, which is a reasonable assumption. An interesting observation is that the optimal values of q and θ do not depend upon the distribution of the waste disposal cost between the manufacturer and the customer. This has to do with the optimal product price $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ charged by the manufacturer. The manufacturer extracts any decrease in the customer's share of the waste disposal cost by charging a higher price. Proposition 2.ii states that the manufacturer's profit decreases linearly in the cost of waste disposal. This is because of two effects. A larger cost of waste disposal decreases the price which the manufacturer can charge to the customer and also increases the manufacturer's share of the waste disposal cost. Proposition 2.iii states that the manufacturer's profit decreases linearly in the environmental cost to the customer during product use. This, again, is because a larger environmental cost depresses the price which the manufacturer can charge to the customer. From Proposition 2.iv we have that the manufacturer's profit increases concavely in the customer's revenue per period from the use of the product. A larger customer revenue from product use enables the manufacturer to charge a higher product price. This alludes to a possible benefit to the manufacturer in sharing effort to help increase customer revenue. The concavity of Π_M with respect to q and θ implies that it could be optimal for the manufacturer to exceed the mandated design standards, if any. Proposition 3 describes the behaviors of the optimal performance and remanufacturability levels with respect to the parameters of the model.

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[15] $\bar{k}_2 = \max\left\{\frac{(u_0+e_0)(c_0q^2-w_0)^2T}{4w_0\phi q(1-\theta)}, \frac{(u_0+e_0)(c_0q^2+w_0)T}{2\phi q}\right\}$.

[16] The reader should bear in mind that q and θ are bounded below by the design standards q_s and θ_s, respectively. Recall that, for expositional convenience and without loss of generality, we set q_s and $\theta_s = 0$.

Proposition 3

i. $\frac{\partial\theta^*}{\partial w_0} > 0$; $\frac{\partial\theta^*}{\partial e_0} > 0$; $\frac{\partial\theta^*}{\partial c_0} > 0$; $\frac{\partial\theta^*}{\partial u_0} > 0$; $\frac{\partial\theta^*}{\partial\phi} < 0$; $\frac{\partial\theta^*}{\partial k_2} < 0$.

ii. $\frac{\partial q^*}{\partial w_0} > 0$; $\frac{\partial q^*}{\partial e_0} > 0$; $\frac{\partial q^*}{\partial c_0} < 0$; $\frac{\partial q^*}{\partial u_0} > 0$; $\frac{\partial q^*}{\partial\phi} < 0$; $\frac{\partial q^*}{\partial k_1} < 0$.

Proof: See Appendix A.

Note again that $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$. θ^* increases in w_0 because of the two effects described earlier. A larger waste disposal cost reduces the price which the manufacturer can charge to the customer for the product and also increases the waste disposal cost incurred by the manufacturer. A larger remanufacturability level reverses these effects. As the coefficient c_0 increases, the manufacturer raises the level of remanufacturability θ^* and lowers performance q^* in order to keep the unit cost of production low. Both θ^* and q^* decrease in the cost of design. This is intuitive since larger design costs should result in lower optimal choices of θ and q. θ^* increases in e_0 and u_0; as the customer faces increasing operating and environmental costs, she replaces the product more frequently, but the optimal price that the manufacturer can charge is correspondingly lower. The manufacturer is thus induced to provide higher remanufacturability in order to reduce both the unit production cost as well as the waste disposal cost. θ^* decreases in ϕ because a larger customer revenue per period increases the price that can be charged by the manufacturer and diminishes the incentive for the manufacturer to reduce the customer's share of the waste disposal cost by providing greater remanufacturability. As w_0 increases, q^* increases in order to decrease u and e and, hence, increase the manufacturer's revenue from selling the product. Note that the customer's optimal replacement interval is $\tau = \frac{\phi}{(u+e)}$. There is an important difference in the manner in which q and θ impact the manufacturer's total revenue over the planning horizon. While r^* increases in both q as well as θ, τ increases in q but is unaffected by θ. Thus, an increase in θ always increases the manufacturer's total revenue over the planning horizon whereas the impact of a change in q on the manufacturer's total revenue over the planning horizon is more intricate. Although not obvious, it is profitable for the manufacturer to increase q and, hence, increase r^* in response to an increase in either e_0 or u_0. q^* decreases in ϕ, again because a larger customer revenue per period increases the price that can be charged by the manufacturer and diminishes the incentive for the manufacturer to reduce the environmental costs during product use by providing greater performance.

Thus, higher waste disposal costs and environmental costs during product use induce higher performance and remanufacturability. Therefore, from an environmental standpoint, it makes sense for the regulator to use these costs to induce environmentally favorable product designs. Empirical evidence of the above is reported in Tojo (2004). As per the Specified Home Appliance Recycling (SHAR) law in Japan, manufacturers and importers of televisions, refrigerators, air conditioners and washing machines are required to take back discarded products, dismantle them and recover components and material that could be reused or recycled; end-users bear the costs of end-of-life management of the products they discard. Typical measures implemented by manufacturers in response to the SHAR law include modular design, component reuse, design for upgradability, reconditioning of products, and remanufacturing. Manufacturers of automobiles in Sweden have responded to legislation on emissions during the product use phase by employing light-weight materials that improve fuel efficiency, such as aluminium, thermoset plastics with glass fibers, and thermoplastics.

However, higher environmental costs result in lower firm profits. A recourse then is to use coordinating contracts, which are known in the literature to lead to higher supply chain profits. Section 1 includes examples from practice of contracts within the context of remanufacturable products. However, outcomes other than profitability have rarely been analyzed in the literature. In the sections to follow, we analyze the case where there is perfect coordination between the manufacturer and the customer; we then contrast the profitability and design outcomes in the uncoordinated and coordinated cases and discuss contracts that can help achieve coordination in the supply chain.

5 Coordinated Case

In this section, we determine the optimal replacement policy and design choices for the integrated firm that optimizes the sum total of the profits of the customer and the manufacturer. Using backward induction again, we first solve for the integrated firm's optimal replacement policy as a function of its design choices. Note that since the firm is integrated, the price decision is absent. We then deduce the firm's optimal design choices of performance and remanufacturability, given the optimal replacement policy.

5.1 Replacement Decision

Since the manufacturer and customer are one integrated entity, the replacement decision is to choose an optimal replacement time interval that maximizes the sum total of the customer's and manufacturer's profits, given design choices of q and θ. Thus, the integrated firm chooses t which maximizes Π_I, where Π_I is given by

$$\Pi_I = \left[\phi - \frac{r+\alpha w}{t} - \frac{(u+e)t}{2}\right] + \left[\frac{r-c}{t} - \frac{\beta w}{t} - \frac{k}{T}\right] = \phi - \frac{(c+w)}{t} - \frac{(u+e)t}{2} - \frac{k}{T} \tag{14}$$

The replacement interval which maximizes Π_I is, therefore,

$$\tau_i = \sqrt{\frac{2(c+w)}{u+e}} \tag{15}$$

Observation 2 lists properties of the optimal replacement interval τ_i with respect to the production cost c, the cost of waste disposal w, the operating cost gradient u, and the environmental cost gradient e.

Observation 2

i. $\frac{\partial \tau_i}{\partial c} > 0$, $\frac{\partial^2 \tau_i}{\partial c^2} < 0$; $\frac{\partial \tau_i}{\partial w} > 0$, $\frac{\partial^2 \tau_i}{\partial w^2} < 0$.

ii. $\frac{\partial \tau_i}{\partial u} < 0$, $\frac{\partial^2 \tau_i}{\partial u^2} > 0$; $\frac{\partial \tau_i}{\partial e} < 0$, $\frac{\partial^2 \tau_i}{\partial e^2} > 0$.

Proof: See Appendix A.

Explanations for the above behaviors are similar to those provided for Observation 1 *mutatis mutandis*. Having solved for the integrated firm's optimal replacement policy, we proceed to the first stage in the sequence of decisions and deduce the firm's optimal design choices of performance and remanufacturability.

5.2 Design Decision

The behavior of the integrated firm's profit function with respect to its design choices of q and θ can be described with the help of the following observations.

$$\frac{\partial \Pi_I}{\partial q} = \frac{-1}{\tau_i}\frac{\partial c}{\partial q} + \left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right]\frac{\partial \tau_i}{\partial q} + \frac{-\tau_i}{2}\frac{\partial u}{\partial q} + \frac{-\tau_i}{2}\frac{\partial e}{\partial q} + \frac{-1}{T}\frac{\partial k}{\partial q} \tag{16}$$

$$\frac{\partial \Pi_I}{\partial \theta} = \frac{-1}{\tau_i}\frac{\partial w}{\partial \theta} + \frac{-1}{\tau_i}\frac{\partial c}{\partial \theta} + \left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right]\frac{\partial \tau_i}{\partial \theta} + \frac{-1}{T}\frac{\partial k}{\partial \theta} \tag{17}$$

We note the following:

i. $\frac{\partial c}{\partial q} > 0$; $\frac{\partial k}{\partial q} > 0$; $\frac{\partial u}{\partial q} < 0$; $\frac{\partial e}{\partial q} < 0$.

ii. $\frac{\partial c}{\partial \theta} < 0$; $\frac{\partial k}{\partial \theta} > 0$; $\frac{\partial w}{\partial \theta} < 0$.

iii. $\frac{\partial \tau_i}{\partial q} = \frac{1}{\sqrt{2(u+e)(c+w)}}\frac{\partial c}{\partial q} - \sqrt{\frac{(c+w)}{2(u+e)^3}}\left[\frac{\partial u}{\partial q} + \frac{\partial e}{\partial q}\right] > 0.$

iv. $\frac{\partial \tau_i}{\partial \theta} = \frac{1}{\sqrt{2(u+e)(c+w)}}\left[\frac{\partial w}{\partial \theta} + \frac{\partial c}{\partial \theta}\right] < 0.$

v. $\left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right] = 0$, since $\tau_i = \sqrt{\frac{2(c+w)}{u+e}}$.

The first term on the right hand side of (16) is < 0 since $\frac{\partial c}{\partial q} > 0$. This term represents the decrease in profit due to a higher unit cost for a superior performance product. The second term on the right hand side of (16) is $= 0$ since $\left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right] = 0$, although $\frac{\partial \tau_i}{\partial q} > 0$. The replacement interval increases in performance because of higher production cost, lower operating costs, and lower environmental costs during product use. However, this effect vanishes because the replacement interval is optimally chosen such that the marginal benefit of decreased operating, environmental, and waste disposal costs from better performance and remanufacturability equates the marginal cost of production. The third term on the right hand side of (16) is > 0 since $\frac{\partial u}{\partial q} < 0$. This term represents the increase in profit due to lower operating costs for a superior performance product. The fourth term on the right hand side of (16) is > 0 since $\frac{\partial e}{\partial q} < 0$. This term represents the increase in profit due to lower environmental costs for a superior performance product. The fifth term on the right hand side of (16) is < 0 since $\frac{\partial k}{\partial q} > 0$. This term represents the decrease in profit due to larger design costs for a superior performance product. The first term on the right hand side of (17) is > 0 since $\frac{\partial w}{\partial \theta} < 0$. This term represents the increase in profit due to a decrease in waste disposal cost with greater remanufacturability. The second term on the right hand side of (17) is > 0 since $\frac{\partial c}{\partial \theta} < 0$. This term represents the increase in profit due to lower production cost for a product with greater remanufacturability. The third term on the right hand side of (17) is $= 0$ since $\left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right] = 0$, although $\frac{\partial \tau_i}{\partial \theta} < 0$. The replacement interval decreases in remanufacturability because of lower production and waste disposal costs. However, this effect vanishes, again because the replacement interval is optimally chosen such that the marginal benefit of decreased operating, environmental, and waste disposal costs from better performance and remanufacturability equates the marginal cost of production. The fourth term on the right hand side of (17) is < 0 since $\frac{\partial k}{\partial \theta} > 0$. This term represents the decrease in profit due to larger design costs for a product with greater remanufacturability. Substituting the optimal replacement interval from

(15) for t in the integrated firm's profit function in (14), we have

$$\Pi_I = \phi - \sqrt{2(c+w)(u+e)} - \frac{k}{T} \tag{18}$$

Proposition 4 provides properties of the integrated firm's profit function with respect to q, θ, and the parameters of the model. We use the same functional forms as those assumed in Section 4.

Proposition 4

i. $\exists\ \bar{k}_1^i,\ \bar{k}_2^i$ such that $\forall\ k_1 > \bar{k}_1^i$ and $k_2 > \bar{k}_2^i$, the integrated firm's profit function is jointly concave in q and θ. (q_i^*, θ_i^*) uniquely maximizes Π_I, where $q_i^* := \left\{q : \frac{\partial \Pi_I}{\partial q} = 0\right\}$ and $\theta_i^* = \max\left\{\theta : \frac{\partial \Pi_I}{\partial \theta} = 0, 0\right\}$.[17]

ii. $\frac{\partial \Pi_I}{\partial w_0} < 0$; $\frac{\partial^2 \Pi_I}{\partial w_0^2} > 0$.

iii. $\frac{\partial \Pi_I}{\partial e_0} < 0$; $\frac{\partial^2 \Pi_I}{\partial e_0^2} > 0$.

iv. $\frac{\partial \Pi_I}{\partial \phi} > 0$; $\frac{\partial^2 \Pi_I}{\partial \phi^2} = 0$.

Proof: See Appendix A.

The optimal values of q_i and θ_i can be derived from the first order conditions $\frac{\partial \Pi_I}{\partial q} = 0$ and $\frac{\partial \Pi_I}{\partial \theta} = 0$. Thus, we have that q_i^* satisfies

$$8k_1^2 q^5 (c_0 q^2 + w_0) - u_0 T^2 (c_0 q^2 - w_0)^2 (1-\theta) = 0 \tag{19}$$

Also, θ_i^* (if > 0) satisfies

$$\theta^3 - \theta^2 + \frac{u_0 T^2 (c_0 q^2 + w_0)}{8 k_2^2 q} = 0 \tag{20}$$

Note that $q_i^* \geq 0$. The left hand side of equation (20) is a polynomial (cubic) function of odd degree with real coefficients. Hence, it has at least one real root. Using *Descartes' Rule of Signs* we have that the above cubic function has one negative root and either two positive real roots or two complex conjugate roots. We require θ_i^* to be a non-negative fraction less than or equal to one. In order that the cubic equation have one negative and two positive roots, we require that the discriminant of the cubic equation be < 0; i.e., we require that $27\xi^2 - 4\xi < 0$, where $\xi = \frac{u_0 T^2 (c_0 q^2 + w_0)}{8 k_2^2 q}$. The assumption $k_2 > \sqrt{\frac{27 u_0 T^2 (c_0 q^2 + w_0)}{32 q}}$ ensures that $27\xi^2 - 4\xi < 0$ and, hence, that the cubic equation has one negative and two positive roots. $\theta \geq 1$ cannot satisfy $\theta^3 - \theta^2 + \xi = 0$. Hence $\theta_i^* > 0$ implies that $\theta_i^* < 1$. Thus, in the coordinated case, the assumptions needed for the integrated firm's profit to be jointly concave in q and θ are that the coefficients of the design costs of performance and remanufacturability should be sufficiently large. These are reasonable assumptions. Propositions 4.ii and 4.iii show that the integrated firm's profit decreases convexly in w_0 and e_0 in contrast to the linear decrease of supply chain profit with respect to w_0 and e_0 in the uncoordinated case. Proposition 4.iv shows that the integrated firm's profit increases linearly with per-period revenue ϕ, in contrast to the concave increase in supply chain profit with respect to ϕ in the

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[17] $\bar{k}_1^i = \max\left\{\sqrt{\frac{(u_0+e_0)(c_0q^2-w_0)^4(1-\theta)T^2}{8q^5(c_0q^2+w_0)^3}},\ \frac{T}{2}\left[\frac{\sqrt{2(u_0+e_0)(1-\theta)}(c_0q^2-w_0)(5c_0q^4+3w_0q^2)}{4\left[q^3(c_0q^2+w_0)\right]^{3/2}} - \frac{c_0\sqrt{2(u_0+e_0)(1-\theta)}}{\sqrt{q(c_0q^2+w_0)}}\right]\right\}$;
$\bar{k}_2^i = \max\left\{\sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)T^2}{8q(1-\theta)^3}},\ \sqrt{\frac{27u_0T^2(c_0q^2+w_0)}{32q}}\right\}$.

uncoordinated case. Thus, the negative effects of increasing environmental costs on supply chain profit are felt more strongly in the uncoordinated case than in the coordinated case. Similar to the uncoordinated case, the concavity of Π_I with respect to q and θ implies that it could be optimal for the integrated firm to exceed the mandated design standards, if any. Proposition 5 describes the behaviors of the optimal performance and remanufacturability levels for the coordinated case, with respect to the parameters of the model.

Proposition 5

i. $\frac{\partial\theta_i^*}{\partial w_0} > 0$; $\frac{\partial\theta_i^*}{\partial e_0} > 0$; $\frac{\partial\theta_i^*}{\partial c_0} > 0$; $\frac{\partial\theta_i^*}{\partial u_0} > 0$; $\frac{\partial\theta_i^*}{\partial \phi} = 0$; $\frac{\partial\theta_i^*}{\partial k_2} < 0$.

ii. $\frac{\partial q_i^*}{\partial w_0} > 0$; $\frac{\partial q_i^*}{\partial e_0} > 0$; $\frac{\partial q_i^*}{\partial c_0} < 0$; $\frac{\partial q_i^*}{\partial u_0} > 0$; $\frac{\partial q_i^*}{\partial \phi} = 0$; $\frac{\partial q_i^*}{\partial k_1} < 0$.

Proof: See Appendix A.

The behavior of the optimal remanufacturability level θ_i^* and the optimal performance level q_i^* with respect to c_0, k_1 and k_2 is intuitive and the explanation for the behavior is similar to that provided for Proposition 3 in Section 4. θ_i^* increases in w_0 in order to lower the cost of waste disposal. From our earlier discussion, we have that $\frac{\partial\tau_i}{\partial\theta} < 0$, and $\frac{\partial\tau_i}{\partial q} > 0$. Any increase in θ increases the frequency of replacements by the firm. Thus, the response to either increasing operating costs (i.e., u_0) or increasing environmental costs during product use (i.e., e_0) is an increase in the frequency of replacements through greater remanufacturability. The response to increasing waste disposal costs (i.e., w_0) is a decrease in the frequency of replacements through better performance. The integrated firm in the coordinated case provides better performance to reduce operating or environmental costs in response to an increase in u_0 or e_0, respectively. The integrated firm's revenues from product use are independent of design choices, and these revenues contribute entirely to the integrated firm's profit. This is in contrast to the uncoordinated case where the customer's revenue from product use, together with q, θ, and r, governs the revenue earned by the manufacturer through the sale of the product to the customer.

6 Comparison between Coordinated and Uncoordinated Cases

In this section, we compare the outcomes of profit and design between the uncoordinated and coordinated cases. Proposition 6 states that the uncoordinated supply chain can never outdo the coordinated chain in terms of profit.

Proposition 6 *The integrated firm's profit Π_I is no less than the total supply chain profit $\Pi_{SC} := \Pi_M + \Pi_C$ in the uncoordinated case.*

Proof: See Appendix A.

The loss to the supply chain from a lack of coordination can be attributed to an analog of the classic double marginalization problem in a decentralized bilateral monopoly (Spengler 1950). In the uncoordinated case, the manufacturer prices the product above cost and this price determines the customer's product replacement frequency. The optimal replacement interval in the uncoordinated case is given by $\sqrt{\frac{2(r+\alpha w)}{u+e}}$, whereas that in the coordinated case is given by $\sqrt{\frac{2(c+w)}{u+e}}$. For the same design choices, the customer in the uncoordinated case replaces the product less frequently than the integrated firm in the coordinated case, since $r + \alpha w \geq c + w$.[18] As will become clearer in our discussion of coordinating contracts in Section 7, in our optimal product replacement situation with remanufacturing, more frequent product replacements are beneficial to the manufacturer because of the more frequent, though smaller revenue earning opportunities. In the uncoordinated case, $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ and $\tau = \frac{\phi}{u+e}$. Therefore, the average revenue per period earned by the manufacturer from product replacements by the customer is $\frac{r^*}{\tau} = \frac{\phi}{2} - \frac{\alpha w(u+e)}{\phi}$, implying that customer revenue from product use does not entirely translate into net supply chain revenue (i.e., $\frac{r^*}{\tau} < \phi$), and also that any increase in the customer's per-period revenue from product use results in a smaller increase in total supply chain profit. A comparison of Propositions 2 and 4 shows that the negative effects of increasing environmental costs on supply chain profit are felt more strongly in the uncoordinated case than in the coordinated case - attributable to the double marginalization effect described above. In addition, while the integrated firm's profit increases linearly with per-period revenue ϕ, the supply chain profit in the uncoordinated case increases only concavely with respect to ϕ. Hence, as regulators expand the use and stringency of EPR instruments, firms have strong incentives to enter into contracts that achieve supply chain coordination. Proposition 7 shows that coordination not only results in higher profits but also drives environmentally superior product design choices.

Proposition 7 *The integrated firm's design choices in the coordinated case are environmentally superior to those in the uncoordinated case.*

Proof: See Appendix A.

Thus, from an environmental standpoint as well, coordination in the supply chain is beneficial. The higher profitability from coordination provides additional incentives for environmentally superior product design choices that alleviate the impacts of environmental costs. The above analytical comparisons provide the basis for Section 7, where we discuss contracts that help achieve coordination in the supply chain - especially since coordination harmonizes the seemingly divergent objectives of environmental benevolence and profitability. A numerical example in Section 8 provides further insights into the results in the uncoordinated and coordinated cases.

7 Coordinating through Contracts

The benefits of supply chain coordination, as well as various mechanisms by which coordination can be achieved, have been well studied in the literature. An excellent review of this literature is provided by Cachon (2003). Several researchers have demonstrated that in integrated supply chains, one partner's gain exceeds the other partner's loss; the net benefit (monetary) can be shared by both in any equitable manner. Our approach to supply chain coordination is along the lines of Monahan (1984), Lee and Rosenblatt (1986), and Weng (1995) who discuss optimal quantity discounts offered by a vendor to induce a favorable ordering pattern by a buyer employing an EOQ policy. The focus in these papers is on vendor and supply chain-optimal order sizes that are larger than privately optimized order sizes by the buyer. In an analogous manner, we focus on the supply chain-optimal product replacement interval by a customer optimizing among product replacement costs, operating costs, and environmental costs during and at the end of the product's economic life. Replacement and waste disposal costs in our model are analogous to setup costs in

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[18] From (6), we require $r \geq c + \beta w$ for the manufacturer to participate.

the traditional EOQ model, while operating costs and environmental costs during product use are analogous to holding costs. We present contracts between the manufacturer and the customer that not only achieve the higher supply chain profit but also the environmentally superior product design of the coordinated case. In the uncoordinated case in Section 4, the distribution of waste disposal costs between the manufacturer and the customer had no impact on the manufacturer's profitability nor did it affect upstream design choices by the manufacturer. It turns out that different allocations of responsibilities for operating and environmental costs during product use also do not have a net impact on the manufacturer's design choices or profitability. The key to achieving supply chain coordination is product replacements at the supply chain-optimal frequency. This can be ensured either by assigning responsibility to the manufacturer for the replacement decision (e.g., contract in Section 7.1) or by inducing the customer to replace the product at the supply chain-optimal frequency (e.g., contracts in Section 7.2).

The contracts described in this section reflect the general concepts of *installed base management* and leasing. Installed base management is an arrangement in which the manufacturer bundles maintenance services along with the sale or lease of its product to the customer. With installed base management in place, the manufacturer takes responsibility for the replacement decision. Xerox is a good example. Xerox proactively maintains photocopying machines at user sites, collects used photocopying machines from user sites, and installs replacements (for example, see *http://www.ccchronicle.com/back/2004_summer/2004-06-18/copier.html*, *"Xerox takes over Columbia's copiers"*). Bhattacharya et al. (2005) provide an excellent discussion of the literature on leasing and installed base management. They compare the policy of selling a product to that of installed base management under different time horizons, under stable and improving technologies for product servicing, under competition, and when remanufacturing is possible across product generations. They find that if the remanufacturing option is considered, installed base management does better than selling. They also provide examples from practice and qualitatively justify the use of leasing and installed base management. However, the existing literature has not studied the impact or potential influence of such arrangements on product design, nor have such arrangements been evaluated in a regulatory context such as EPR. A major contribution of our paper lies in demonstrating that, under EPR legislation, coordination in the supply chain through such arrangements can lead not only to higher supply chain profit but also to environmentally superior product design. An important point to note is that the analysis in this section applies quite generally, without the need to assume specific functional forms for k, c, u, e, and w. Also, we do not discuss how the additional profit from coordination should be split between the manufacturer and the customer, since any of the approaches discussed in prior literature (e.g., Weng (1995)) can easily be applied. Table 1 summarizes the contracts examined in this section.

7.1 Coordinating Contract when Manufacturer makes Replacement Decision

In this contract, either the customer or the manufacturer bears the operating and environmental costs during product use but the manufacturer assumes responsibility for the replacement interval decision.[19] The sequence of decisions as per this contract is as follows. The manufacturer first chooses q and θ jointly. He then chooses the replacement interval t for the product followed by the price $r > c$ to be charged to the

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[19] Note, however, that the contract can coordinate the supply chain for *any* distribution of operating *and/or* environmental costs during product use between the manufacturer and the customer.

customer for product replacements. The customer buys replacements at price r if she makes her reservation profit given q, θ, t and r. The customer's profit is given by

$$\Pi_C \quad = \quad \phi - \frac{(r+\alpha w)}{t} - \delta\frac{(u+e)t}{2} \tag{21}$$

where t is the replacement interval chosen by the manufacturer. $\delta = 1$ if the operating and environmental costs during product use are borne by the customer, and $\delta = 0$ if the manufacturer bears these costs. Without loss of generality, the participation constraint for the customer is $\Pi_C \geq 0$, or

$$\phi t - r - \alpha w - \delta\frac{(u+e)t^2}{2} \geq 0 \tag{22}$$

The optimization problem for the manufacturer subject to the customer's participation constraint in (22) is

$$\max_{(q,\theta),t,r} \Pi_M = \frac{(r-c)}{t} - \frac{\beta w}{t} - (1-\delta)\frac{(u+e)t}{2} - \frac{k}{T} \tag{23}$$

Given design choices of q and θ, and the replacement interval t, the manufacturer's profit increases in the price $r > c$ charged to the customer. Hence, it is optimal for the manufacturer to price the product at $r_C^* = \phi t - \alpha w - \delta\frac{(u+e)t^2}{2}$. Substituting r_C^* for r in the manufacturer's profit function in (23), we have

$$\Pi_M = \phi - \frac{(c+w)}{t} - \frac{(u+e)t}{2} - \frac{k}{T} \tag{24}$$

Π_M is concave in t. Hence, $\tau_i = \sqrt{\frac{2(c+w)}{u+e}} = t : \frac{\partial \Pi_M}{\partial t} = 0$ uniquely maximizes Π_M. Substituting τ_i for t in (24), we have

$$\Pi_M = \phi - \sqrt{2(c+w)(u+e)} - \frac{k}{T} \tag{25}$$

which is identical to the integrated firm's profit function in (18) that is optimized with respect to q and θ. In the first stage of the sequence of decisions under this contract, the manufacturer optimizes its profit in (25) with respect to q and θ. This implies that the resulting optimal values of q and θ are identical to those in the coordinated case. Hence, a contract in which either the customer or the manufacturer bears the operating and environmental costs during product use while the manufacturer assumes responsibility for the replacement interval decision, achieves the higher supply chain profit as well as the environmentally superior product design of the coordinated case. Recall that in the uncoordinated case analyzed in Section 4, the manufacturer prices the product at $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$. Therefore, using (8), we have[20]

$$r^* - r_C^* = \frac{\phi^2}{2(u+e)} - \phi\sqrt{\frac{2(c+w)}{u+e}} - (c+w) = \frac{\left[\phi - \sqrt{2(c+w)(u+e)}\right]^2}{2(u+e)} \geq 0 \tag{26}$$

In other words, as per the coordinating contract, the manufacturer provides a price discount in order for the customer to accept more frequent replacements, analogous to the *all units quantity discount* price schedule outlined in Dolan (1987) and Goyal & Gupta (1989). The net effect is that the manufacturer's average revenue per period from sales of product replacements as per the coordinating contract is higher than that

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[20] Note that the inequality in (26) is strict for non-zero choices of either q or θ.

Table 1: Summary of Contracts Examined

Contract type	Replacement decision by	Operating costs borne by	Contract coordinates
Manufacturer makes Replacement Decision	Manufacturer	Either	Profit, Design
Two-Part Tariff (r, F)	Customer	Customer	Profit, Design
Leasing: Menu $\{f, t_f\}$	Customer	Customer	Profit, Design

in the uncoordinated case. Notice that

$$
\begin{aligned}
\frac{r_C^*}{\tau_i} - \frac{r^*}{\tau} &= \frac{\phi}{2} - \frac{\alpha w\sqrt{u+e}}{\sqrt{2(c+w)}} + \frac{\alpha w(u+e)}{\phi} \\
&\geq \frac{\sqrt{2(c+w)(u+e)}}{2} - \frac{\alpha w\sqrt{u+e}}{\sqrt{2(c+w)}} + \frac{\alpha w(u+e)}{\phi}, \quad \text{using (8)} \\
&> 0.
\end{aligned}
$$

7.2 Coordinating Contracts when Customer makes Replacement Decision

In situations where it is not possible for the manufacturer to make the replacement decision, other contracts can be used to achieve supply chain coordination. We discuss two such contracts below.

7.2.1 Two-Part Tariff

In this contract, the manufacturer decides a schedule (r,F), where r is the price the customer pays per replacement instance and F is a fixed fee for the duration of the planning horizon. The customer bears the operating and environmental costs during product use and chooses the optimal replacement interval that maximizes her profit. The sequence of decisions is as follows. The manufacturer first chooses q and θ jointly and then decides the schedule (r,F) that maximizes his profit, subject to the customer's participation constraint. The customer's profit is given by

$$\Pi_C = \phi - \frac{(r+\alpha w)}{t} - \frac{F}{T} - \frac{(u+e)t}{2}$$

The manufacturer's problem, subject to the customer's participation constraint $\Pi_C \geq 0$, is

$$\max_{(q,\theta),(r,F)} \Pi_M = \frac{(r-c)}{t} + \frac{F}{t} - \frac{\beta w}{t} - \frac{k}{T} \tag{27}$$

It is easy to show that, at optimality, the manufacturer chooses the price $r^* = c + \beta w$ and the fixed fee $F^* = [\phi - \sqrt{2(c+w)(u+e)}]T$. In other words, the manufacturer prices the product at the effective marginal cost and extracts all of the customer's profits over the planning horizon via the fixed fee (also see Jeuland & Shugan 1983, Dolan 1987). The customer's incentives are thus aligned with those of the supply chain and the customer chooses the supply chain-optimal replacement interval $\tau_i = \sqrt{\frac{2(c+w)}{u+e}}$. Thus, the manufacturer's design choices of q and θ correspond to those of the integrated firm.

7.2.2 Leasing

Starting with Coase (1972), many researchers have examined the implications of selling versus leasing a durable product, for the manufacturer's pricing and distribution strategies. Most of this literature concerns the *time inconsistency* problem conjectured by Coase - a durable goods monopolist has an incentive to reduce his selling price in future selling periods in order to successively skim the market; rational customers who anticipate such behavior might render pricing above marginal cost in earlier periods impossible. In this context, leasing can be used by the monopolist to increase his profit by overcoming the time inconsistency problem (see Bulow 1982, Stokey 1981). The implications of leasing and selling have also been examined in the contexts of optimal choice of product durability by monopolists (Bulow 1986) and oligopolists (Desai & Purohit 1999), and incentives to innovate over time (Waldman 1996). However, research to date has not examined the implications of leasing when manufacturers are legally responsible for product take-back and life-cycle environmental costs, which create incentives for the manufacturer to invest in product design in order to make products environmentally more favorable. Consider the following sequence of decisions. The manufacturer first chooses q and θ jointly. He then chooses a menu $\{f, t_f\}$ that specifies combinations of per-period product usage fees and corresponding replacement intervals. The customer bears the operating and environmental costs during product use; she chooses a combination of f and t_f from the menu such that her profit is maximized. The customer's profit is given by

$$\Pi_C \quad = \quad \phi - f - \frac{\alpha w}{t_f} - \frac{(u+e)t_f}{2} \tag{28}$$

where f is the per-period fee charged by the manufacturer and t_f is the corresponding replacement interval required by the manufacturer. The optimization problem for the manufacturer (in the absence of a customer participation constraint) is

$$\max_{(q,\theta),\{f,t_f\}} \Pi_M = f - \frac{c}{t_f} - \frac{\beta w}{t_f} - \frac{k}{T} \tag{29}$$

In order that the resulting design choices of q and θ be identical to those in the coordinated case, we require that the manufacturer's profit function maximized with respect to q and θ be identical to the integrated firm's profit function in (18). Thus, the design-coordinating combinations of the per-period fee and the replacement interval that constitute the menu, are

$$\{f, \tau_f\} = \left\{ f, \frac{c+\beta w}{f - \phi + \sqrt{2(c+w)(u+e)}} \right\} \tag{30}$$

We require $f > \phi - \sqrt{2(c+w)(u+e)}$ in order that $\tau_f > 0$. It can be shown that the customer's profit is concave in f. Hence, the customer chooses $f : \frac{\partial \Pi_C}{\partial f} = 0$, subject to her participation constraint $\Pi_C \geq 0$ being met. Thus, the customer chooses $f^* = \phi - [c + (1+\alpha)w]\sqrt{\frac{u+e}{2(c+w)}}$ and $\tau_f^* = \tau_i = \sqrt{\frac{2(c+w)}{u+e}}$ from the menu. In other words, the customer chooses a combination of the fee and replacement interval that coordinates product design *and* supply chain profit.

8 A Numerical Example

Tables 2 and 3 provide numerical illustrations of the results for the uncoordinated and coordinated cases and facilitate further comparisons between the results for the two cases. Note that the values for ϕ, e, $\Pi_{SC} = \Pi_M + \Pi_C$, and Π_I are per-period amounts whereas the values for w and r^* are amounts per replacement instance. As seen in Tables 2 and 3, the optimal reaction to increasing w_0 is, for the

Table 2: Numerical Illustration: Uncoordinated Case

Scenario†	w_0	w	e_0	e	q^*	θ^*	τ	Π_{SC}	r^*
Increasing w_0	200	186.18	20	152.73	0.1309	0.0691	2.07	590.86	1457.63
$\phi = 1500$	400	359.37	20	118.04	0.1694	0.1016	2.68	514.09	1826.78
	600	523.19	20	102.11	0.1959	0.1280	3.09	452.81	2057.87
Increasing w_0	200	191.28	20	171.15	0.1169	0.0436	3.08	1144.18	3748.32
$\phi = 2500$	400	374.07	20	133.02	0.1504	0.0648	3.96	1090.63	4758.95
	600	550.69	20	115.26	0.1735	0.0822	4.57	1047.22	5432.75
Increasing e_0	400	363.21	10	60.49	0.1653	0.0920	2.92	533.51	2006.27
$\phi = 1500$	400	359.37	20	118.04	0.1694	0.1016	2.68	514.09	1826.78
	400	355.53	30	173.29	0.1731	0.1112	2.47	495.07	1677.04
Increasing e_0	400	376.43	10	68.39	0.1462	0.0589	4.30	1103.53	5187.43
$\phi = 2500$	400	374.07	20	133.02	0.1504	0.0648	3.96	1090.63	4758.95
	400	371.72	30	194.68	0.1541	0.0707	3.67	1078.00	4400.44

†$c_0 = 5000, u_0 = 75, k_1 = 75000, k_2 = 50000, \alpha = \beta = 0.5, T = 50.$

manufacturer in the uncoordinated case and the integrated firm in the coordinated case, to provide greater performance as well as remanufacturability. From Table 2, we see that with $w_0 = 400$ and $e_0 = 20$, when the customer's revenue per period increases from 1500 to 2500, the manufacturer can charge a price which is 160.51% higher. However, the incentives for the manufacturer to provide performance and remanufacturability decrease when the customer's per-period revenue increases. From Table 2, we see that when $w_0 = 400$ and $e_0 = 20$, the drop in performance is 11.22% and the drop in remanufacturability is 36.22% when the customer's per-period revenue increases from 1500 to 2500. However, in the coordinated case, design choices are unaffected by changes in the integrated firm's per-period revenue. As seen in Table 3, design choices remain exactly the same even if revenue per period decreases from 2500 to 1500.

Notice that the total supply chain profit in both the uncoordinated and the coordinated cases decreases with increasing waste disposal costs (i.e., increasing w_0) or increasing environmental costs during product use (i.e., increasing e_0). For the uncoordinated case in Table 2, the 1000 unit increase in customer revenue from 1500 to 2500 per period results in a correspondingly smaller increase in supply chain profit in absolute terms; for example, when $w_0 = 400$ and $e_0 = 20$, this increase in supply chain profit is 576.54. In contrast, for the coordinated case in Table 3, any increase in revenue from product use results in an identical increase in supply chain profit. In addition, the integrated firm's profit in the coordinated case always exceeds the corresponding supply chain profit in the uncoordinated case.

The monetary impact of the lack of coordination decreases as the cost of waste disposal increases (i.e., as w_0 increases). The increase in supply chain profit as a result of coordination is 46.29% when $\phi = 1500$, $w_0 = 200$ and $e_0 = 20$, but is 38.37% when $\phi = 1500$, $w_0 = 600$ and $e_0 = 20$. The corresponding increases are 62.94% and 55.32% when $\phi = 2500$. In addition, in Table 2, when $w_0 = 200$ and $e_0 = 20$, the increase in supply chain profit in absolute terms when customer revenue per period increases from 1500 to 2500, is 553.32 whereas the increase in supply chain profit is 594.41 when $w_0 = 600$ and $e_0 = 20$. The monetary impact of the lack of coordination decreases in the environmental cost during product use (i.e., in e_0) too. The increase in supply chain profit as a result of coordination is 42.72% when $\phi = 1500$, $w_0 = 400$ and $e_0 = 10$, and is 39.50% when $\phi = 1500$, $w_0 = 400$ and $e_0 = 30$. The corresponding increases are 59.62% and 56.83% when $\phi = 2500$. In addition, in Table 2, when $w_0 = 400$ and $e_0 = 10$, the increase in supply

Table 3: Numerical Illustration: Coordinated Case

Scenario[‡]	w_0	w	e_0	e	q_i^*	θ_i^*	τ_i	Π_I	Π_I/Π_{SC}	q_i^*/q^*	θ_i^*/θ^*
Increasing w_0	200	165.57	20	129.33	0.1546	0.1722	0.93	864.37	1.46	1.18	2.49
$\phi = 1500$	400	314.66	20	101.53	0.1970	0.2134	1.39	724.93	1.41	1.16	2.10
	600	453.46	20	89.15	0.2243	0.2442	1.74	626.54	1.38	1.15	1.91
Increasing w_0	200	165.57	20	129.33	0.1546	0.1722	0.93	1864.37	1.63	1.32	3.95
$\phi = 2500$	400	314.66	20	101.53	0.1970	0.2134	1.39	1724.93	1.58	1.31	3.29
	600	453.46	20	89.15	0.2243	0.2442	1.74	1626.54	1.55	1.29	2.97
Increasing e_0	400	319.79	10	51.29	0.1950	0.2006	1.47	761.42	1.43	1.18	2.18
$\phi = 1500$	400	314.66	20	101.53	0.1970	0.2134	1.39	724.93	1.41	1.16	2.10
	400	309.70	30	150.95	0.1987	0.2258	1.32	690.64	1.40	1.15	2.03
Increasing e_0	400	319.79	10	51.29	0.1950	0.2006	1.47	1761.42	1.60	1.33	3.41
$\phi = 2500$	400	314.66	20	101.53	0.1970	0.2134	1.39	1724.93	1.58	1.31	3.29
	400	309.70	30	150.95	0.1987	0.2258	1.32	1690.64	1.57	1.29	3.19

[‡] $c_0 = 5000, u_0 = 75, k_1 = 75000, k_2 = 50000, \alpha = \beta = 0.5, T = 50.$

chain profit when customer revenue per period increases from 1500 to 2500 is 570.02 whereas the increase in supply chain profit is 582.93 when $w_0 = 400$ and $e_0 = 30$.

An important point to note is that optimal design choices in the coordinated case are always more environmentally favorable than the corresponding optimal design choices in the uncoordinated case. The divergence between the optimal design choices in the two cases, however, decreases as either w_0 or e_0 increases. When $\phi = 1500, w_0 = 200, e_0 = 20$, the optimal performance level in the coordinated case is 18.11% higher than that in the uncoordinated case. The difference is 14.50% when $\phi = 1500, w_0 = 600, e_0 = 20$. Corresponding differences are 32.25% and 29.28% when $\phi = 2500$. When $\phi = 1500, w_0 = 400, e_0 = 10$, the optimal performance level in the coordinated case is 17.97% higher than that in the uncoordinated case. The difference is 14.79% when $\phi = 1500, w_0 = 400, e_0 = 30$. Corresponding differences are 33.38% and 28.94% when $\phi = 2500$. Likewise, when $\phi = 1500, w_0 = 200, e_0 = 20$, the optimal remanufacturability level in the coordinated case is 149.20% higher than that in the uncoordinated case. The difference is 90.78% when $\phi = 1500, w_0 = 600, e_0 = 20$. Corresponding differences are 294.95% and 197.08% when $\phi = 2500$. When $\phi = 1500, w_0 = 400, e_0 = 10$, the optimal remanufacturability level in the coordinated case is 118.04% higher than that in the uncoordinated case. The difference is 103.06% when $\phi = 1500, w_0 = 400, e_0 = 30$. Corresponding differences are 240.58% and 219.38% when $\phi = 2500$. The differences in the optimal performance levels and in the optimal remanufacturability levels between the uncoordinated and coordinated cases widen as ϕ increases. This is because optimal design choices are invariant with respect to per-period revenue in the coordinated case, in contrast to the uncoordinated case where design choices are environmentally more unfavorable when the customer's per-period revenue is greater.

9 Discussion and Future Work

EPR has significant implications for management theory and practice. It necessitates the incorporation of life-cycle considerations into product-related decisions - including design, procurement, and manufacturing, and requires closer coordination between upstream and downstream actors in the supply chain (Stoughton et al. 1999). As a substantial step in exploring the managerial implications of EPR, we analytically establish optimal product design decisions by a manufacturer producing and selling a remanufacturable product, in response to various implementations of EPR. To explore the potential benefits of contractual arrangements, we consider two distinct supply chain structures - the *uncoordinated* case and the *coordinated case.* Our work thus interfaces with the literatures on closed-loop supply chains and supply chain contracts under the increasingly salient context of environmental regulation in the form of EPR. A novel contribution of our work to the literature on closed-loop supply chains lies in the explicit incorporation of environmental legislation into strategic managerial decisions. In addition, our work reinforces the supply chain contracting literature that looks primarily at profitability outcomes, by demonstrating that coordination can lead not only to higher supply chain profit but also to environmentally favorable product design. The explicit connection between the manufacturer's product design choices and demand for the product via cost-optimal product replacements by the customer is also novel in our model.

In both the uncoordinated and the coordinated cases we find that the optimal levels of performance and remanufacturability increase in the cost of waste disposal and in the environmental cost during product use. Therefore, from an environmental standpoint, it makes sense for the regulator to use these costs to induce environmentally favorable product designs. However, higher environmental costs result in lower firm profits and are therefore likely to meet resistance from industry. A recourse then is to use coordinating contracts, which are known in the literature to lead to higher supply chain profits. Importantly, we show that from an environmental standpoint as well, coordination in the supply chain is advantageous. The contracts presented in Section 7 help achieve supply chain coordination.

Our results provide insights to firms subject to or anticipating EPR legislation, as well as to regulators implementing or contemplating EPR legislation. From a managerial perspective, we present a methodology to incorporate environmental regulation and customer replacement behavior into strategic product design decisions. In addition, the research effort addresses the impacts of environmental policy parameters on upstream environmental design choices by the manufacturer. An increase in the waste disposal cost achieves a reduction in the amount of waste disposed of by inducing a higher level of remanufacturability that increases recovery after product use, and also by inducing a higher level of performance that decreases the frequency of product replacements. The distribution of waste disposal costs between the manufacturer and the customer has no net impact on design outcomes. The manufacturer in the uncoordinated case trades off revenue from the sale of the product in each replacement instance against the frequency of product replacements by the customer. Although not obvious, it is profitable for the manufacturer in the uncoordinated case and the integrated firm in the coordinated case to increase performance in response to an increase in the environmental cost during product use. We also find that it could be optimal for the manufacturer to go beyond mandated design standards, if any. In order for design standards to have bite, it is important for the regulator to understand what the firm would optimally do in the absence of design standards. Thus, the impacts that EPR instruments have on design outcomes are intricate and not obvious. Coordination in the supply chain better meets the profitability objective of the firm as well as the environmental design goals of the regulator. In addition, the negative impacts of increasing environmental costs on profitability are felt more strongly in the uncoordinated case than in the coordinated case. Hence, as regulators expand the use and stringency of EPR instruments, firms have strong incentives to enter into contracts that achieve supply chain coordination. The seemingly divergent objectives of environmental benevolence and profitability can be harmonized through the coordinating contracts described in Section

7. Importantly, these contracts are very generally applicable, irrespective of the specific functional forms assumed for the purpose of analysis. The suggested contracts reflect the general concepts of leasing and installed base management.

Several extensions to this work merit treatment in future research, though we believe that our basic qualitative insights would largely continue to hold. In assuming that remanufacturing is profitable to the manufacturer and is also preferred by the knowledgable customer, we abstracted from the cannibalization effect that remanufactured products have on the sales of new products. Also, we assumed that a single manufacturer sells to a single customer. The effects of considering multiple customers and heterogeneous preferences with regard to new and remanufactured products, are worth evaluating. In addition, competition between manufacturers for customer demand can change upstream outcomes in interesting ways in the presence of customers' switching costs. Information asymmetry between the manufacturer and the customer with regard to the manufacturer's production costs and the customer's operating costs could be studied within the context of our model. Uncertainty could be incorporated into the model in different ways. There could be uncertainty in the success of design efforts, in the possibility that a returned product could be remanufactured, and in environmental policies over time. Uncertainty and information asymmetry will render further complexity to the analysis of coordinating contracts. Finally, additional insights into EPR policy design can be facilitated by endogenizing the policy parameters of the model.

Appendix A: Proofs

Proof of Observation 1:

i. $\frac{\partial\tau}{\partial r} = \frac{1}{\sqrt{2(u+e)(r+\alpha w)}} > 0, \frac{\partial^2\tau}{\partial r^2} = -\frac{1}{2}\frac{1}{\sqrt{2(u+e)(r+\alpha w)^3}} < 0.$ ∎

$\frac{\partial\tau}{\partial w} = \frac{\alpha}{\sqrt{2(u+e)(r+\alpha w)}} > 0, \frac{\partial^2\tau}{\partial w^2} = -\frac{\alpha^2}{2\sqrt{2(u+e)(r+\alpha w)^3}} < 0.$ ∎

$\frac{\partial\tau}{\partial \alpha} = \frac{w}{\sqrt{2(u+e)(r+\alpha w)}} > 0;, \frac{\partial^2\tau}{\partial \alpha^2} = -\frac{w^2}{2\sqrt{2(u+e)(r+\alpha w)^3}} < 0.$ ∎

ii. $\frac{\partial\tau}{\partial u} = -\frac{1}{2}\sqrt{\frac{2(r+\alpha w)}{(u+e)^3}} < 0, \frac{\partial^2\tau}{\partial u^2} = \frac{3}{4}\sqrt{\frac{2(r+\alpha w)}{(u+e)^5}} > 0.$ ∎

$\frac{\partial\tau}{\partial e} = -\frac{1}{2}\sqrt{\frac{2(r+\alpha w)}{(u+e)^3}} < 0, \frac{\partial^2\tau}{\partial e^2} = \frac{3}{4}\sqrt{\frac{2(r+\alpha w)}{(u+e)^5}} > 0.$ ∎

Proof of Observation 2:

i. $\frac{\partial\tau_i}{\partial c} = \frac{1}{\sqrt{2(u+e)(c+w)}} > 0, \frac{\partial^2\tau_i}{\partial c^2} = -\frac{1}{2}\frac{1}{\sqrt{2(u+e)(c+w)^3}} < 0.$ ∎

$\frac{\partial\tau_i}{\partial w} = \frac{1}{\sqrt{2(u+e)(c+w)}} > 0, \frac{\partial^2\tau_i}{\partial w^2} = -\frac{1}{2\sqrt{2(u+e)(c+w)^3}} < 0.$ ∎

ii. $\frac{\partial\tau_i}{\partial u} = -\frac{1}{2}\sqrt{\frac{2(c+w)}{(u+e)^3}} < 0, \frac{\partial^2\tau_i}{\partial u^2} = \frac{3}{4}\sqrt{\frac{2(c+w)}{(u+e)^5}} > 0.$ ∎

$\frac{\partial\tau_i}{\partial e} = -\frac{1}{2}\sqrt{\frac{2(c+w)}{(u+e)^3}} < 0, \frac{\partial^2\tau_i}{\partial e^2} = \frac{3}{4}\sqrt{\frac{2(c+w)}{(u+e)^5}} > 0.$ ∎

Proof of Proposition 1:

$$\begin{aligned}
\Pi_M &= \frac{r-c-\beta w}{\sqrt{\frac{2(r+\alpha w)}{u+e}}} - \frac{k}{T} \\
\frac{\partial \Pi_M}{\partial r} &= \frac{(r+c+2\alpha w+\beta w)\sqrt{u+e}}{2\sqrt{2}(r+\alpha w)^{3/2}} > 0 \\
\frac{\partial^2 \Pi_M}{\partial r^2} &= -\frac{(r+3c+4\alpha w+3\beta w)\sqrt{u+e}}{4\sqrt{2}(r+\alpha w)^{5/2}} < 0. \quad ∎
\end{aligned}$$

Proof of Proposition 2:

i. $\frac{\partial \Pi_M}{\partial q} = \frac{(w_0 - c_0 q^2)(u_0+e_0)(1-\theta)T - 2k_1\phi q^3}{\phi q^2 T}$; $\frac{\partial^2 \Pi_M}{\partial q^2} = -\frac{2[(u_0+e_0)w_0(1-\theta)T + k_1\phi q^3]}{\phi q^3 T} < 0.$
$\frac{\partial \Pi_M}{\partial \theta} = \frac{c_0(u_0+e_0)q^2T + (u_0+e_0)w_0T - 2k_2\phi q\theta}{\phi q T}$; $\frac{\partial^2 \Pi_M}{\partial \theta^2} = -\frac{2k_2}{T} < 0.$
The determinant of the Hessian matrix of $\Pi_M(q,\theta)$
$= \frac{-(u_0+e_0)^2(c_0q^2-w_0)^2T^2 + 4k_2\phi(u_0+e_0)w_0q(1-\theta)T + 4k_1k_2\phi^2q^4}{\phi^2q^4T^2} > 0$ if $k_2 > \frac{(u_0+e_0)(c_0q^2-w_0)^2T}{4w_0\phi q(1-\theta)}$.
$\forall\ k_2 > \bar{k}_2$, the Hessian matrix of Π_M with respect to q and θ is negative definite, yielding the result. $\blacksquare$

ii. $\frac{\partial \Pi_M}{\partial w_0} = -\frac{(u_0+e_0)(1-\theta)}{\phi q} < 0$; $\frac{\partial^2 \Pi_M}{\partial w_0^2} = 0.$ $\blacksquare$

iii. $\frac{\partial \Pi_M}{\partial e_0} = -\frac{(c_0q^2+w_0)(1-\theta)}{\phi q} < 0$; $\frac{\partial^2 \Pi_M}{\partial e_0^2} = 0.$ $\blacksquare$

iv. $\frac{\partial \Pi_M}{\partial \phi} = \frac{\phi^2 + 2(u+e)(c+w)}{2\phi^2} > 0$; $\frac{\partial^2 \Pi_M}{\partial \phi^2} = -\frac{2(u+e)(c+w)}{\phi^3} < 0.$ $\blacksquare$

Proof of Proposition 3:

i. $\frac{\partial \theta^*}{\partial w_0} = \frac{(u_0+e_0)T}{2k_2\phi q} > 0.$ $\blacksquare$
$\frac{\partial \theta^*}{\partial e_0} = \frac{(c_0q^2+w_0)T}{2k_2\phi q} > 0.$ $\blacksquare$
$\frac{\partial \theta^*}{\partial c_0} = \frac{(u_0+e_0)qT}{2k_2\phi} > 0.$ $\blacksquare$
$\frac{\partial \theta^*}{\partial u_0} = \frac{(c_0q^2+w_0)T}{2k_2\phi q} > 0.$ $\blacksquare$
$\frac{\partial \theta^*}{\partial \phi} = -\frac{(u_0+e_0)(c_0q^2+w_0)T}{2k_2\phi^2 q} < 0.$ $\blacksquare$
$\frac{\partial \theta^*}{\partial k_2} = -\frac{(u_0+e_0)(c_0q^2+w_0)T}{2k_2^2\phi q} < 0.$ $\blacksquare$

ii. Denote $f_1 := \frac{\partial \Pi_M}{\partial q}$. q^* satisfies $\frac{\partial \Pi_M}{\partial q} = 0$; i.e., $f_1(q^*) = 0$. Using the *Implicit Function Theorem*, we have $\frac{\partial q^*}{\partial x} = -\frac{(\partial f_1/\partial x)}{(\partial f_1/\partial q^*)}$. Since Π_M is concave in q, $(\partial f_1/\partial q^*) < 0$. Hence, $\frac{\partial q^*}{\partial x} = \frac{(\partial f_1/\partial x)}{|\partial f_1/\partial q^*|}$.
$\frac{\partial q^*}{\partial w_0} = \frac{(u_0+e_0)(1-\theta)q^*T}{2[(u_0+e_0)(1-\theta)w_0T + k_1\phi q^{*3}]} > 0.$ $\blacksquare$
$\frac{\partial q^*}{\partial e_0} = \frac{(w_0-c_0q^{*2})(1-\theta)q^*T}{2[(u_0+e_0)(1-\theta)w_0T + k_1\phi q^{*3}]} > 0$, since $w_0 > c_0q^2$ from Assumption A6. $\blacksquare$
$\frac{\partial q^*}{\partial c_0} = -\frac{(u_0+e_0)(1-\theta)q^{*3}T}{2[(u_0+e_0)(1-\theta)w_0T + k_1\phi q^{*3}]} < 0.$ $\blacksquare$
$\frac{\partial q^*}{\partial u_0} = \frac{(w_0-c_0q^{*2})(1-\theta)q^*T}{2[(u_0+e_0)(1-\theta)w_0T + k_1\phi q^{*3}]} > 0.$ $\blacksquare$
$\frac{\partial q^*}{\partial \phi} = \frac{(u_0+e_0)(c_0q^{*2}-w_0)(1-\theta)q^*T}{2\phi[(u_0+e_0)(1-\theta)w_0T + k_1\phi q^{*3}]} < 0.$ $\blacksquare$
$\frac{\partial q^*}{\partial k_1} = -\frac{\phi q^{*4}}{(u_0+e_0)(1-\theta)w_0T + k_1\phi q^{*3}} < 0.$ $\blacksquare$

Proof of Proposition 4:

i. $\frac{\partial \Pi_I}{\partial q} = \frac{\sqrt{(u_0+e_0)(1-\theta)}(w_0-c_0q^2)}{\sqrt{2q^3(c_0q^2+w_0)}} - \frac{2k_1q}{T}$; $\frac{\partial^2 \Pi_I}{\partial q^2} = -\frac{c_0\sqrt{2(u_0+e_0)(1-\theta)}}{\sqrt{q(c_0q^2+w_0)}} + \frac{\sqrt{2(u_0+e_0)(1-\theta)}(c_0q^2-w_0)(5c_0q^4+3w_0q^2)}{4[q^3(c_0q^2+w_0)]^{3/2}} - \frac{2k_1}{T} < 0$ if $k_1 > \frac{T}{2}\left[\frac{\sqrt{2(u_0+e_0)(1-\theta)}(c_0q^2-w_0)(5c_0q^4+3w_0q^2)}{4[q^3(c_0q^2+w_0)]^{3/2}} - \frac{c_0\sqrt{2(u_0+e_0)(1-\theta)}}{\sqrt{q(c_0q^2+w_0)}}\right]$.
$\frac{\partial \Pi_I}{\partial \theta} = \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)}{2q(1-\theta)}} - \frac{2k_2\theta}{T}$; $\frac{\partial^2 \Pi_I}{\partial \theta^2} = \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)}{8q(1-\theta)^3}} - \frac{2k_2}{T} < 0$ if $k_2 > \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)T^2}{32q(1-\theta)^3}}$.
The determinant of the Hessian matrix of $\Pi_I(q,\theta)$
$= \frac{-(u_0+e_0)^2(1-\theta)(c_0q^2+w_0)^2w_0T^2 + 4\sqrt{2q}k_2\big[(u_0+e_0)(1-\theta)(c_0q^2+w_0)\big]^{3/2}(1-\theta)w_0T + 8k_1k_2(u_0+e_0)(c_0q^2+w_0)^2(1-\theta)^2q^3}{2(u_0+e_0)(c_0q^2+w_0)^2(1-\theta)^2q^3T^2}$
$- \frac{\sqrt{2(u_0+e_0)^3(c_0q^2+w_0)(1-\theta)q}\big[k_1q^2(c_0q^2+w_0)^2 + k_2(1-\theta)^2(c_0q^2-w_0)^2\big]T}{2(u_0+e_0)(c_0q^2+w_0)^2(1-\theta)^2q^3T^2} > 0$ if $k_1 > \sqrt{\frac{(u_0+e_0)(c_0q^2-w_0)^4(1-\theta)T^2}{8q^5(c_0q^2+w_0)^3}}$ and $k_2 > \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)T^2}{8q(1-\theta)^3}}$. Note that the lower bound on θ is 0. $\forall\ k_1 > \bar{k}_1^i$ and $k_2 > \bar{k}_2^i$, the Hessian matrix of Π_I with respect to q and θ is negative definite, yielding the result. $\blacksquare$

ii. $\frac{\partial \Pi_I}{\partial w_0} = -\sqrt{\frac{(u_0+e_0)(1-\theta)}{2q(c_0q^2+w_0)}} < 0$; $\frac{\partial^2 \Pi_I}{\partial w_0^2} = \sqrt{\frac{(u_0+e_0)(1-\theta)}{8q(c_0q^2+w_0)^3}} > 0.$ $\blacksquare$

iii. $\frac{\partial \Pi_I}{\partial e_0} = -\sqrt{\frac{(c_0q^2+w_0)(1-\theta)}{2q(u_0+e_0)}} < 0$; $\frac{\partial^2 \Pi_I}{\partial e_0^2} = \sqrt{\frac{(c_0q^2+w_0)(1-\theta)}{8q(u_0+e_0)^3}} > 0.$ $\blacksquare$

iv. $\frac{\partial \Pi_I}{\partial \phi} = 1$; $\frac{\partial^2 \Pi_I}{\partial \phi^2} = 0.$ $\blacksquare$

Proof of Proposition 5:

i. Denote $f_2 := \frac{\partial \Pi_I}{\partial \theta}$. θ_i^* satisfies $\frac{\partial \Pi_I}{\partial \theta} = 0$; i.e., $f_2(\theta_i^*) = 0$. Using the Implicit Function Theorem, we have $\frac{\partial \theta_i^*}{\partial x} = -\frac{(\partial f_2/\partial x)}{(\partial f_2/\partial \theta_i^*)}$. Since Π_I is concave in θ, $(\partial f_2/\partial \theta_i^*) < 0$. Hence, $\frac{\partial \theta_i^*}{\partial x} = \frac{(\partial f_2/\partial x)}{\left|\partial f_2/\partial \theta_i^*\right|}$.
   $\frac{\partial f_2}{\partial w_0} = \sqrt{\frac{(u_0+e_0)}{8q(w_0+c_0q^2)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial w_0} > 0.$ ▮
   $\frac{\partial f_2}{\partial e_0} = \sqrt{\frac{(w_0+c_0q^2)}{8q(u_0+e_0)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial e_0} > 0.$ ▮
   $\frac{\partial f_2}{\partial c_0} = \sqrt{\frac{(u_0+e_0)q^3}{8(w_0+c_0q^2)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial c_0} > 0.$ ▮
   $\frac{\partial f_2}{\partial u_0} = \sqrt{\frac{(w_0+c_0q^2)}{8q(u_0+e_0)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial u_0} > 0.$ ▮
   $\frac{\partial f_2}{\partial \phi} = 0 \Rightarrow \frac{\partial \theta_i^*}{\partial \phi} = 0.$ ▮
   $\frac{\partial f_2}{\partial k_2} = -\frac{2\theta_i^*}{T} < 0 \Rightarrow \frac{\partial \theta_i^*}{\partial k_2} < 0.$ ▮

ii. Denote $f_3 := \frac{\partial \Pi_I}{\partial q}$. q_i^* satisfies $\frac{\partial \Pi_I}{\partial q} = 0$; i.e., $f_3(q_i^*) = 0$. Using the Implicit Function Theorem, we have $\frac{\partial q_i^*}{\partial x} = -\frac{(\partial f_3/\partial x)}{(\partial f_3/\partial q_i^*)}$. Since Π_I is concave in q, $(\partial f_3/\partial q_i^*) < 0$. Hence, $\frac{\partial q_i^*}{\partial x} = \frac{(\partial f_3/\partial x)}{\left|\partial f_3/\partial q_i^*\right|}$.
   $\frac{\partial f_3}{\partial w_0} = \frac{(3c_0{q_i^*}^2+w_0)\sqrt{(u_0+e_0)(1-\theta)}}{\sqrt{8{q_i^*}^3(w_0+c_0{q_i^*}^2)^3}} > 0 \Rightarrow \frac{\partial q_i^*}{\partial w_0} > 0.$ ▮
   $\frac{\partial f_3}{\partial e_0} = \frac{(w_0-c_0{q_i^*}^2)\sqrt{1-\theta}}{\sqrt{8{q_i^*}^3(u_0+e_0)(w_0+c_0{q_i^*}^2)}} \Rightarrow \frac{\partial q_i^*}{\partial e_0} > 0$, since $w_0 > c_0q^2$ from Assumption A6. ▮
   $\frac{\partial f_3}{\partial c_0} = -\frac{(c_0{q_i^*}^2+3w_0)\sqrt{q_i^*(u_0+e_0)(1-\theta)}}{\sqrt{8(w_0+c_0{q_i^*}^2)^3}} < 0 \Rightarrow \frac{\partial q_i^*}{\partial c_0} < 0.$ ▮
   $\frac{\partial f_3}{\partial u_0} = \frac{(w_0-c_0{q_i^*}^2)\sqrt{1-\theta}}{\sqrt{8{q_i^*}^3(u_0+e_0)(w_0+c_0{q_i^*}^2)}} \Rightarrow \frac{\partial q_i^*}{\partial u_0} > 0.$ ▮
   $\frac{\partial f_3}{\partial \phi} = 0 \Rightarrow \frac{\partial q_i^*}{\partial \phi} = 0.$ ▮
   $\frac{\partial f_3}{\partial \phi} = -\frac{2q_i^*}{T} < 0 \Rightarrow \frac{\partial q_i^*}{\partial k_1} < 0.$ ▮

Proof of Proposition 6:

From (11), we have

$$\Pi_{SC} = \Pi_M + \Pi_C = \frac{\phi^2 - 2(c+w)(u+e)}{2\phi} - \frac{k}{T}$$

And from (18), we have

$$\Pi_I = \phi - \sqrt{2(c+w)(u+e)} - \frac{k}{T}$$

Therefore, for the same design choices,

$$\begin{aligned}\Pi_I - \Pi_{SC} &= \frac{\phi}{2} - \sqrt{2(c+w)(u+e)} + \frac{(c+w)(u+e)}{\phi} \\ &= \frac{\left[\phi - \sqrt{2(c+w)(u+e)}\right]^2}{2\phi} \\ &\geq 0, \text{ using (8).} \quad ▮ \end{aligned} \tag{31}$$

Note that (7) and, hence, (8) and (31) are strict inequalities for non-zero choices of either q or θ, since k_1, $k_2 > 0$.

Proof of Proposition 7:

Since Π_M and Π_I are both jointly concave in q and θ, it suffices to show that $\frac{\partial \Pi_I}{\partial q} \geq \frac{\partial \Pi_M}{\partial q}$, and that $\frac{\partial \Pi_I}{\partial \theta} \geq \frac{\partial \Pi_M}{\partial \theta}$. From the proofs of Propositions 2 and 4 we have

$$\begin{aligned}\frac{\partial \Pi_M}{\partial q} &= \frac{(w_0 - c_0q^2)(u_0+e_0)(1-\theta)}{\phi q^2} - \frac{2k_1q}{T} \\ \text{and } \frac{\partial \Pi_I}{\partial q} &= \frac{\sqrt{(u_0+e_0)(1-\theta)}(w_0 - c_0q^2)}{\sqrt{2q^3(w_0+c_0q^2)}} - \frac{2k_1q}{T} \\ \text{Hence, } \frac{\partial \Pi_I}{\partial q} - \frac{\partial \Pi_M}{\partial q} &= \frac{\sqrt{(u_0+e_0)(1-\theta)}(w_0 - c_0q^2)}{\phi\sqrt{2q^3(w_0+c_0q^2)}}\left[\phi - \sqrt{2(c+w)(u+e)}\right] \\ &\geq 0, \text{ using (8) and since } w_0 > c_0q^2 \text{ from Assumption A6.} \quad ▮ \end{aligned} \tag{32}$$

And,

$$\begin{aligned}
\frac{\partial \Pi_M}{\partial \theta} &= \frac{(w_0 + c_0 q^2)(u_0 + e_0)}{\phi q} - \frac{2k_2\theta}{T} \\
\text{and } \frac{\partial \Pi_I}{\partial \theta} &= \frac{\sqrt{(w_0 + c_0 q^2)(u_0 + e_0)}}{\sqrt{2q(1-\theta)}} - \frac{2k_2\theta}{T} \\
\text{Hence, } \frac{\partial \Pi_I}{\partial \theta} - \frac{\partial \Pi_M}{\partial \theta} &= \frac{\sqrt{(w_0 + c_0 q^2)(u_0 + e_0)}}{\phi\sqrt{2q(1-\theta)}} \left[\phi - \sqrt{2(c+w)(u+e)}\right] \\
&\geq 0, \text{ using (8).} \quad \blacksquare \qquad (33)
\end{aligned}$$

Note, again, that the inequalities in (32) and (33) are strict for non-zero choices of either q or θ.

Appendix B: EPR Instruments from Practice

The definition of EPR according to Lindhqvist (1992) is as follows:[21]

> Extended Producer Responsibility is an environmental protection strategy to reach an environmental objective of a decreased total environmental impact from a product, by making the manufacturer of the product responsible for the entire life-cycle of the product and especially for the take-back, recycling and final disposal of the product. Extended Producer Responsibility is implemented through administrative, economic and informative instruments. The composition of these instruments determines the precise form of Extended Producer Responsibility.

EPR instruments fall into three broad and possibly overlapping categories - *take-back requirements*, *economic instruments*, and *performance standards* (OECD 2001). Take-back requirements assign responsibilities to the beneficiaries of products for end-of-life product management. Economic instruments such as deposit/refund systems, advance disposal fees, and material taxes are incentive-based and provide flexibility in establishing the means to accomplish the EPR target. Performance standards, such as minimum recycled content, can be set to specify a particular percentage of materials to be recovered and reused. Examples of EPR instruments from practice include:[22]

i. *Product Take-Back* and/or *Recovery Targets*: Specified Home Appliance Recycling (SHAR) Law in Japan mandating recovery/reuse targets in terms of product weight; Ordinance on Producer Responsibility for Cars in Sweden making manufacturers and importers of cars in Sweden responsible for accepting end-of-life vehicles; German Packaging Ordinance mandating recycling/reuse targets for product packaging; Used Oil/Containers/Filters Industry Management Program of Western Canada for the collection and processing of used oil, oil containers and oil filters.

ii. *Deposit/Refund Systems:* Deposit/Refund Systems for beverage containers, batteries, and tires in certain states in the US; Deposit/Refund Systems for food containers, tires, batteries, lubricants, pesticide containers, and plastics in South Korea.

iii. *Advance Disposal Fees:* Advance disposal fees charged by manufacturers facing the SHAR law to consumers of white goods in Japan; Paint Stewardship Program in British Columbia where "eco-fees" are charged to customers at the point of sale.

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[21] The concept of EPR was first introduced by Thomas Lindhqvist of the IIIEE (International Institute for Industrial Environmental Economics) to the Swedish Ministry of the Environment in 1990.

[22] Sources: OECD (2005), Tojo (2004), US Environmental Protection Agency (*http://www.epa.gov/otaq/mpg.htm*), UK Department for Environment, Food and Rural Affairs (*http://www.defra.gov.uk/environment/climatechange/trading/eu/*), New Zealand Business Council for Sustainable Development (*http://www.nzbcsd.org.nz/story.asp?id=13*), *http://www.environment-agency.gov.uk/business/444304/444641/595811/136872/?lang=_e*, *http://www.colby.edu/personal/t/thtieten/Dep.htm*, *http://www.publications.parliament.uk/pa/cm199900/cmbills/043/2000043.htm*, *http://www.ilsr.org/recycling/epr/tools.html*, *http://www.green-alliance.org.uk*.

iv. *Product Design Standards:* Recycled Content of Newsprint Bill in the UK specifying minimum recycled content for newsprint; European Directive on Packaging and Packaging Waste which limits concentration levels of lead, cadmium, mercury and hexavalent chromium in packaging or packaging components[23]; Corporate Average Fuel Economy (CAFE) standards in the US requiring vehicle manufacturers to comply with fuel economy standards set by the Department of Transportation.

v. *Costs/Charges for Environmental Impact during Product Use*: Costs for Carbon Dioxide emissions incurred by firms in the European Union subject to the EU Greenhouse Gas Emissions Trading Program; Gas Guzzler Tax in the US, imposed on manufacturers on the sale of cars failing to meet fuel economy standards.

vi. *Material Taxes*: “Eco-tax” on PVC in Belgium and Denmark aimed at shifting consumption away from PVC.

vii. *Other Measures*: In addition to the above instruments, a variety of measures have been implemented to complement and support the goals of EPR policies and programs. Examples include eco-labelling, green procurement, and product stewardship.

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[23] Over thirty-five states in the US have enacted some type of product or packaging restrictions. For example, Rhode Island prohibits non-biodegradable plastic carrier rings, packaging containing potentially toxic heavy metals, metal beverage containers with detachable flip tops, plastic food or beverage containers composed of more than one resin, degradable plastic containers which interfere with recycling, and telephone directory binders which interfere with recycling. See *http://www.ilsr.org/recycling/epr/tools.html.*

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